Orthogonal Time Frequency Space (OTFS) Modulation Based Radar System

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Abstract—Orthogonal time frequency space (OTFS) modulation was proposed to tackle the destructive Doppler effects in wireless communications, with potential applications to many other areas. In this paper, we investigate its application to radar systems, and propose a novel efficient OTFS-based matched filter algorithm for target range and velocity estimation. The proposed algorithm not only exhibits the inherent advantages due to multi-carrier modulation of the existing orthogonal frequency division multiplexing (OFDM-) based radar algorithms but also provides additional benefits to improve radar capability. Similar to OFDM, OTFS spreads the transmitted signal in the entire time–frequency resources to exploit the full diversity gains for radar processing. However, OTFS requires less cyclic prefix, and hence, shorter transmission duration than OFDM, allowing longer range radar and/or faster target tracking rate. Additionally, unlike OFDM, OTFS is inter-carrier interference-free, enabling larger Doppler frequency estimation. We demonstrate the performance of the proposed algorithm using numerical results under different system settings.

Index Terms—Delay–Doppler channel, matched filter, OTFS, OFDM, radar systems.

I. INTRODUCTION

The concept of using a waveform to perform simultaneously the tasks of radar and communications (i.e., so-called RadCom) has found wide range of applications, both in modern civilian and commercial areas. For example, in emerging intelligent transportation applications, RadCom systems offer both communication links to other vehicles and active surrounding environment sensing functionalities, enabling cooperative interactions between all vehicles on the road [1]. Other applications of RadCom systems are unsurprisingly found in aeronautical and military areas. However, despite much research, there remain limitations of the current RadCom (and radar) systems to support emerging applications such as intelligent automotive systems with high mobility and dense traffic environment requiring very high data rate, ultra reliability, and ultra low latency communications. Under these conditions, it is challenging to develop suitable waveforms to simultaneously satisfy the requirements for both radar sensing and data communications.

Orthogonal frequency division multiplexing (OFDM) modulation has been considered in existing literature for RadCom applications [2]–[7]. Despite its many advantages such as simple random signal generation, full digital processing, and high processing gains in time and frequency, OFDM based waveforms exhibit drawbacks in radar sensing such as Doppler intolerance [8]. More critically, they also suffer from heavy degradation in data communications under high Doppler environments such as high-speed railway mobile communications. A more elaborate analysis on the interference in OFDM can be found in [17]–[19].

Recently, in [9], [10], the authors proposed orthogonal time frequency space (OTFS) modulation for communications over multi-path delay–Doppler channels where each path exhibits a different delay and Doppler shift. The delay–Doppler domain provides as an alternative representation of a time-varying channel geometry due to moving objects (e.g., transmitters, receivers, or reflectors) in the scene. Leveraging on this representation, OTFS multiplexes each information symbol over a two dimensional (2D) orthogonal basis functions, specifically designed to combat the dynamics of time-varying multipath channels. Then the information symbols placed in the delay-Doppler coordinate system can be converted to the standard time-frequency domain used by traditional multi-carrier modulation schemes such as OFDM.

OTFS was shown to provide significant error performance advantages over OFDM over delay–Doppler channels with a wide range of Doppler frequencies [9]–[16]. With a suitable message passing based OTFS detection algorithm [13], the performance of OTFS is in general independent of Doppler frequencies for a given pulse shape unlike OFDM. These results inspire the potential applications of OTFS in many fields beyond wireless communications. In this paper, we investigate such an application of OTFS in RadCom and radar systems. While OTFS based waveforms are suitable for data communications, especially in high-mobility environments such as self-driving cars, high-speed trains, drones, flying cars, and supersonic flights, it remains to be studied whether OTFS is well placed for radar processing (i.e., object angle, range, and velocity estimation), which is the focus of this paper.

More specifically, this paper proposes a novel OTFS-based matched filter algorithm to estimate the range and velocity of objects in radar systems. The proposed radar technique is motivated from our previous results on embedded channel estimation algorithms for OTFS [14], [15]. Instead of embedding a known (or pilot) data symbol in an OTFS frame for transmit channel estimation, random equal power data symbols occupying a full OTFS frame are used for radar processing. By applying a simple matched filter-based algorithm on the
received signal at the transmitter, we can efficiently estimate the number of potential targets as well as their corresponding ranges and velocities. The proposed algorithm retains the inherent advantages of multi-carrier modulation to exploit the full diversity gains for radar processing since OTFS spreads the transmitted signal in the entire time–frequency resources. Moreover, to transmit the same number of symbols, OTFS requires less cyclic prefix (CP), and hence, shorter transmission duration than OFDM, allowing longer range radar capability and/or faster target tracking rate. Another attractive property of OTFS-based radar is that it is inter-carrier interference (ICI)-free, enabling larger Doppler frequency estimation. OFDM-based radar algorithms suffer larger ICI under higher Doppler frequencies due to the loss of orthogonality across sub-carriers, preventing the detection of larger Doppler frequencies (i.e., high-mobility targets). In general, while OFDM can exactly prevent the detection of larger Doppler frequencies (i.e., free, enabling larger Doppler frequency estimation. OFDM-and/or faster target tracking rate. Another attractive property of duration than OFDM, allowing longer range radar capability the transmitted signal in the entire time–frequency resources.

In this section, we describe the signal model for OTFS-based radar following [9], [10], [13].

The rest of the paper is organized as follows. Section II describes OTFS-based radar signal model, which lay the foundations for the development of OTFS-based matched filter radar algorithm in Section III. Numerical results are presented in Section IV followed by the conclusions in Section V.

II. OTFS-BASED RADAR

In this section, we describe the signal model for OTFS-based radar following [9], [10], [13].

A. Basic OTFS concepts/notations

- The time–frequency signal plane is discretized to a M by N grid (for some integers N, M > 0) by sampling the time and frequency axes at intervals of T (seconds) and \( \Delta f = 1/T \) (Hz), respectively, i.e.,

\[ \Lambda = \{(nT, m\Delta f), \ n = 0, \ldots, N - 1, \ m = 0, \ldots, M - 1\} \]

- The modulated time–frequency samples \( X[n, m], n = 0, \ldots, N - 1, \ m = 0, \ldots, M - 1 \) are transmitted over an OTFS frame with duration \( T_f = NT \) and occupy a bandwidth \( B = M\Delta f \).

- The delay–Doppler plane in the region \((0, T) \times (-\Delta f/2, \Delta f/2)\) is discretized to an M by N grid

\[ \Gamma = \left\{ \left( \frac{k}{NT}, \frac{l}{M\Delta f} \right), \ k = 0, \ldots, N-1, l = 0, \ldots, M-1 \right\} \]

where \(1/M\Delta f\) and \(1/NT\) represent the quantization steps of the delay and Doppler frequency axes, respectively.

B. OTFS-based radar signal model

The modulator first maps a set of \( NM \) random symbols \( \{x[k, l], k = 0, \ldots, N - 1, l = 0, \ldots, M - 1\} \) from a modulation alphabet \( A = \{a_1, \ldots, a_Q\} \) (e.g. QAM symbols) of size \( Q \), arranged on the delay–Doppler domain grid \( \Gamma \), to samples \( X[n, m] \) in the time–frequency domain grid using the inverse symplectic finite Fourier transform (ISFFT). Next, the Heisenberg transform is applied to \( X[n, m] \) using transmit pulse \( g_{tx}(t) \) to create the continuous-time radar signal \( s(t) \), which is then transmitted over the multi-path delay–Doppler radar channel.

Remark 1: Compared to OFDM, OTFS has superior peak-to-average-power ratio (PAPR) properties thanks to the ISFFT as [9], [10]. However, it is still not a constant modulus waveform, which would be more appropriate for the automotive radar applications.

Assume there are \( P \) targets (or objects), each with range \( R_i \) and relative velocity \( V_i, i = 1, \ldots, P \) relative to the transmitter. Note that \( V_i \) can be positive or negative. Denote:

\[ \frac{\tau_i}{2} = \frac{R_i}{c} \quad \frac{\nu_i}{2} = f_c \frac{V_i}{c} \]

where \( f_c \) is the carrier frequency, \( c \) is the speed of light, \( \tau_i \) is the round-trip delay between the transmitter and the \( i \)-th target, \( \nu_i \) is the Doppler frequency of the \( i \)-th target. We can express the delay–Doppler radar channel \( h(\tau, \nu) \) as:

\[ h(\tau, \nu) = \sum_{i=1}^{P} h_i \delta(\tau - \tau_i)\delta(\nu - \nu_i), \]

where \( h_i \) denotes the complex gain of the \( i \)-th target, and \( \delta(\cdot) \) denotes the Dirac delta function.

As the grid dimension of the delay–Doppler plane is \( M \times N \), the delay and Doppler ranges that can be detected in OTFS are given by \( (0, \frac{M-1}{\Delta f}) \) and \( (-\frac{1}{2T}, \frac{1}{2T}) \), respectively. In this work, for simplicity, we assume the delay and Doppler of the targets are
integer multiples of the delay resolution $\frac{1}{N\Delta T}$ and Doppler resolution $\frac{1}{M\Delta f}$, respectively.\(^1\) Therefore, the delay–Doppler radar channel $h(\tau, \nu)$ in (1) will be expressed as:

$$ h(\tau, \nu) = \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} h[k, l] \delta \left( \tau - \frac{l}{M\Delta f} \right) \delta \left( \nu - \frac{(k)N}{NT} \right) $$

(2)

where

$$(k)N = \begin{cases} k & \text{if } k \leq N/2 \\ k - N & \text{otherwise} \end{cases}$$

and $h[k, l]$ denotes the complex gain of the target at the Doppler tap $k$ and delay tap $l$ corresponding to Doppler frequency $\frac{k}{NT}$ and delay $\frac{l}{M\Delta f}$. If there is not such target with Doppler tap $k$ and delay tap $l$ then $h[k, l] = 0$.

The time-domain signal $r(t)$ received at the transmitter after being reflected from the targets can be expressed as:

$$ r(t) = \int \int h(\tau, \nu)s(t-\tau)e^{j2\pi\nu(t-\tau)}d\tau d\nu. $$

(3)

$r(t)$ is processed by the Wigner transform (implementing a receiver filter with an impulse response $g_{\text{rx}}(t)$) followed by a sampler, to produce the received samples $y[n, m]$ in the time–frequency domain. We then apply SFFT on $y[n, m]$ to obtain the received symbols $y[k, l]$ in the delay–Doppler domain for radar processing. In this paper, we assume $g_{\text{tx}}(t)$ and $g_{\text{tx}}(t)$ as rectangular pulses. Note that the relation in (3) is valid for the wideband signals $s(t)$.

We now look at the relations between received symbols $y[k, l]$ and transmitted symbols $x[k, l]$. The exact relation between $y[k, l]$ and $x[k, l]$ was derived in [12] as

$$ y[k, l] = \sum_{k'=0}^{N-1} \sum_{l'=0}^{M-1} h[k', l']e^{j2\pi\left(\frac{\nu 1}{2M}\right)} \alpha_{i}[k, l] $$

$$ x[k - k', N, l - l', M] + w[k, l], $$

(4)

where

$$ \alpha_{i}[k, l] = \begin{cases} 1 & 0 \leq l < M \\ e^{j2\pi\frac{k'}{M}} & \text{ otherwise} \end{cases} $$

and $w[k, l] \sim \mathcal{CN}(0, \sigma^2)$ is additive white noise with variance $\sigma^2$, $[\cdot]_N$ and $[\cdot]_M$ denote modulo $N$ and $M$ operations, respectively. We can see that the output–input relation in OTFS is similar to a 2D circular convolution except for an additional phase shift that depends on the location of symbols in the grid.

III. OTFS-BASED RADAR MATCHED FILTER ALGORITHM

In the OTFS-based radar system as in Figure 1, we know the transmit symbols $x[k, l]$ and the received symbols $y[k, l]$. However, we do not know the delay–Doppler radar channel $h(\tau, \nu)$ in (1) or (2). The purpose of radar processing is to estimate $h(\tau, \nu)$, which will provide us the information on the targets and corresponding ranges and velocities.

We now propose a matched filter based method to estimate $h(\tau, \nu)$, which is equivalent to detect range and velocity of the targets, i.e., $h[k, l]$ for $0 \leq k \leq N - 1, 0 \leq l \leq M - 1$.

The input–output relation in (4) can be vectorized as

$$ y = \mathbf{H}x + w, $$

(5)

where $\mathbf{H} \in \mathbb{C}^{MN \times MN}$ and the $(k + Ni)$-th elements of $y, x, w \in \mathbb{C}^{MN \times 1}$ are equal to $y[k, l], x[k, l]$, and $w[k, l]$, respectively. Then, we write (5) in an alternative form as

$$ y = \tilde{\mathbf{X}}h + w, $$

(6)

where the $(k + Ni)$-th element of $h$ is $h[k, l]$ and the $(i, j)$-th element of $\tilde{\mathbf{X}} \in \mathbb{C}^{MN \times MN}$, $0 \leq i = k' + Ni', 0 \leq j = k'' + Ni'' \leq M - 1$ is given in (7). Note that the first column of $\tilde{\mathbf{X}}$, i.e., $k'' = l'' = 0$, is equal to $x$ and all the remaining columns of $\tilde{\mathbf{X}}$ are the circular shifts of $x$ with a particular phase shift.

In order to obtain the estimate of $h$ from (6), we propose the following matched filter (MF) detection:

$$ \tilde{h} = \mathbf{X}^H y = \mathbf{X}^H \mathbf{X} \tilde{h} + \mathbf{X}^H w = \mathbf{G}h + \tilde{w} $$

(8)

where the gain matrix $\mathbf{G} = \mathbf{X}^H \mathbf{X} \in \mathbb{C}^{MN \times MN}$ and $\tilde{w} = \mathbf{X}^H w$.\(^2\)

Assume the symbols $x[k, l], 0 \leq k \leq N - 1, 0 \leq l \leq M - 1$ are drawn as independently and identically distributed (i.i.d.) quadrature phase shift keying (QPSK) symbols of power $P_s$. We can now examine the properties of the gain matrix $\mathbf{G}$.

1) From the circulant property of $\mathbf{X}$, the $i$-th diagonal element of $\mathbf{G}$, $\mathbf{G}[i, i], 0 \leq i \leq MN - 1$, reduces to

$$ \mathbf{G}[i, i] = \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} |x[k, l]|^2 = MN P_s $$

(9)

2) The off-diagonal elements of $\mathbf{G}$, $\mathbf{G}[i, j], 0 \leq i \neq j \leq MN - 1$, can be written as

$$ \mathbf{G}[i, j] = \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x^*[k - k', N, l - l', M] \phi(i, j, k, l) $$

(10)

where $i = k' + Ni', j = k'' + Ni''$ and

$$ \phi(i, j, k, l) = \begin{cases} e^{j2\pi\frac{\nu 1}{M}} & \text{if } l' \leq l < l'' \\ e^{j2\pi\frac{k'}{M}} & \text{if } l \leq l' \leq l'' \end{cases} $$

(11)

where $\phi(i, j, k, l) = e^{j2\pi\frac{(k'N)(l'' - l') + (k''N)(l - l')}{MN}}$. Now, the mean and variance of $\mathbf{G}[i, j]$ is given in the following lemma.

**Lemma 1**: The mean and variance of $\mathbf{G}[i, j]$ are as follows

$$ \text{E}[\mathbf{G}[i, j]] = 0 $$

$$ \text{var}[\mathbf{G}[i, j]] = MN P_s^2 $$

(12)

(13)

**Proof**: See Appendix.

Hence, from (9) and Lemma 1, we can clearly see that as $MN$
Parameter & Value
\begin{tabular}{|c|c|}
\hline
Symbol & Value \\
\hline
$N_c$ & Carrier frequency 24 GHz \\
$N_s$ & Number of subcarriers 256 \\
$B$ & Total signal bandwidth 10 MHz \\
$\Delta f$ & Subcarrier spacing 39.063 KHz \\
$N_e$ & Number of evaluated symbols 64 \\
$\Delta R$ & Range resolution 15 m \\
$\Delta V$ & Velocity resolution 3.8125 m/s \\
$R_{max}$ & Unambiguous range 3840 m \\
$V_{max}$ & Unambiguous velocity $\pm 122$ m/s \\
SNR & Signal-to-noise ratio ($P_s/\sigma^2$) 10 dB \\
\hline
\end{tabular}

A. Advantages of OTFS-based radar over OFDM-based radar

OTFS-based radar processing has advantages over OFDM-based radar processing in two aspects namely time resources and high Doppler interference.

1) While OTFS-based radar system requires one CP to transmit all the $NM$ symbols, OFDM-based radar system \cite{1} requires $N$ CP’s. Therefore, OTFS radar saves a total of $(N-1)L$ symbols transmission time, which is attractive particularly for the targets at the longer ranges. The reduction in transmission time also allows to detect the target more frequently compared to OFDM, which provides faster tracking rates.

2) Under larger Doppler frequencies, OFDM system experience higher ICI due to the loss of orthogonality across subcarriers, limiting OFDM capability to estimate the larger Doppler frequencies. In general, OFDM can accurately estimate the Doppler frequencies only up to 10% of the subcarrier spacing ($\Delta f$). However, OTFS can estimate the Doppler frequencies up to $\Delta f$ without any interference.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, we demonstrate the superior performance of OTFS over OFDM through numerical results. For OFDM simulations, we consider the conventional FFT method proposed in \cite{1}. The summary of the simulation parameters is given in Table I. Here, we assume a single reflecting target with unit channel amplitude, i.e., $h_1 = 1$. Note that OTFS uses a total transmit power of $(NM+(L-1))P_s$, whereas OFDM requires a total transmit power of $(NM+(L-1)N)P_s$, where the value of $L$ depends on the maximum range to be detected. In the simulations, we analyze the performance in terms of peak-to maximum sidelobe ratio (PSLR) and root mean squared error (RMSE) at different values of signal to noise ratio (SNR).

Figures 3 and 4 show the normalized range and velocity profile for the single reflecting object with $R = 975$ m and $V = 80$ m/s. It can be observed that OFDM is able to detect the range of the target without any error, but experiences an error of 19 m/s for the velocity. On the other hand, OTFS is able to detect both the range and velocity without any errors. Moreover, PSLR is higher in OTFS compared to OFDM. The error in velocity and low PSLR of OFDM can be explained by the high ICI due to high Doppler ($65\%$ of $\Delta f$).

Figure 5 displays RMSE of the velocity estimates for different target relative velocities with $R = 975$ m. We assume the target velocities as the integer multiples of velocity resolution, $\Delta V$ and consider 100 Montecarlo simulations. We can see that while the RMSE of OFDM increases with relative velocity (> 25% error at $\pm 90$ m/s), OTFS experience zero RMSE, which is useful to detect high Doppler targets. Note that the proposed

\[
\mathbf{X}[i,j] = \begin{cases} 
    x[i[(k'-k)N],[l'-l]M]e^{-j2\pi \frac{(k'-(l'-l)M)}{MN} \sigma^2} & \text{if } l' < l'' \\
    x[i[(k'-k)N],[l'-l]M]e^{j2\pi \frac{(k'-(l'-l)M)}{MN} \sigma^2} & \text{otherwise}
\end{cases}
\]
The PSLR and image SNR behaviour can also be interpreted using noise-limited and interference-limited regions. For lower SNR = $P_s/\sigma^2$ values, the sidelobes are in the noise-limited region where the noise power dominates over the sidelobes, the image SNR and PSLR increase almost linearly with SNR. Once the symbol SNR increases, i.e., sidelobe power is higher than noise (interference-limited region), PSLR and image SNR saturates into approximately the time-bandwidth product (see (13)). The saturation values of PSLR and image SNR’s could be further improved by implementing more efficient detection algorithms, which we will consider in future work.

V. CONCLUSION

We have proposed a novel efficient orthogonal time frequency space based matched filter algorithm for object range and velocity determination in radar systems. We show that OTFS-based radar processing not only exhibits the inherent advantages due to multi-carrier modulation but also provides additional benefits for improved radar capability, such as longer range, faster tracking rate, as well as larger Doppler frequency estimation compared to the popular orthogonal frequency division multiplexing (OFDM) based radar. Our results demonstrate that OTFS-based radar with adequate detection algorithms is a promising robust technique to detect the long range and high velocity targets.

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APPENDIX

PROOF OF LEMMA 1

Mean of $G[i, j]$ – From (10), the value of $\mathbb{E}[G[i, j]]$ can be simplified to zero as in from (14) to (16). Here, we invoke the i.i.d. property of $x[k, l]$ as $(k', l') \neq (k'', l'')$ for off-diagonal elements.
\[
\mathbb{E}[G[i, j]] = \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \mathbb{E}[x^*([k-k']_N, [l-l']_M)x([k-k'']_N, [l-l'']_M)] \phi(i, j, k, l)
\]
(14)

\[
= \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \mathbb{E}[x^*([k-k']_N, [l-l']_M)]\mathbb{E}[x([k-k'']_N, [l-l'']_M)] \phi(i, j, k, l)
\]
(15)

\[
= 0
\]
(16)

\[
\text{var}[G[i, j]] = \sum_{k_1=0}^{N-1} \sum_{l_1=0}^{M-1} \sum_{k_2=0}^{N-1} \sum_{l_2=0}^{M-1} \mathbb{E}[x^*([k_1-k']_N, [l_1-l']_M)x([k_2-k'']_N, [l_2-l'']_M)] \phi(i, j, k_1, l_1)
\]
(17)

\[
= \sum_{k_1=0}^{N-1} \sum_{l_1=0}^{M-1} \mathbb{E}[x([k_1-k']_N, [l_1-l']_M)]^2 \mathbb{E}[x([k_2-k'']_N, [l_2-l'']_M)]^2 + \sum_{k_1=0}^{N-1} \sum_{l_1=0}^{M-1} \sum_{k_2=0}^{N-1} \sum_{l_2=0}^{M-1} \mathbb{E}[x^*([k_1-k']_N, [l_1-l']_M)] \mathbb{E}[x([k_2-k'']_N, [l_2-l'']_M)] \phi(i, j, k_1, l_1) \phi^*(i, j, k_2, l_2)
\]
(18)

\[
= MNP_s^2 + \sum_{k_1=0}^{N-1} \sum_{l_1=0}^{M-1} \sum_{k_2=0}^{N-1} \sum_{l_2=0}^{M-1} F_{ij}(k_1, l_1, k_2, l_2) \phi(i, j, k_1, l_1) \phi^*(i, j, k_2, l_2) = MNP_s^2 + F_{ij}
\]
(19)

\[
F_{ij}(k_1, l_1, k_2, l_2) = \mathbb{E}[x^2([k_1-k']_N, [l_1-l']_M)]\mathbb{E}[x^2([k_1-k'']_N, [l_1-l'']_M)]
\]
(20)

\[
= 0
\]
(21)

Variance of $G[i, j]$ – From (10) and (16), the value of $\text{var}[G[i, j]]$ can be written as (17). The value of (17) can be split into sum of $MN P_s^2$ and $F_{ij}$ as in (18) and (19). The assumption $(k', l') \neq (k'', l'')$ is used in (18) and the i.i.d. property of $x[k, l]$ is used in (19).

Now, the value of $F_{ij}(k_1, l_1, k_2, l_2)$ may be non-zero only if the following four conditions are satisfied:

\[
[k_1 - k']_N = [k_2 - k'']_N, [l_1 - l']_M = [l_2 - l'']_M
\]
\[
[k_1 - k'']_N = [k_2 - k']_N, [l_1 - l']_M = [l_2 - l']_M
\]

In these cases, the value of $F_{ij}(k_1, l_1, k_2, l_2)$ can be simplified to zero as shown in (20–21), which concludes the proof.

References


