Millimeter Wave Analog Beamforming With Low Resolution Phase Shifters for Multiuser Uplink

P. Raviteja, *Student Member, IEEE*, Yi Hong, *Senior Member, IEEE*, and Emanuele Viterbo, *Fellow, IEEE*

***Abstract*—We consider millimeter wave multiuser uplink system with low resolution phase shifters, where users transmit simultaneously to base station. Transmit and receive beamforming through large antenna arrays is used to compensate severe path loss of millimeter waves. We first propose a joint precoder and detector design based on the low-complexity local search algorithm that iteratively finds the preferred transmit and receive beamforming vectors, which maximizes the sum-rate of the multiuser uplink system. Although the joint design achieves similar sum-rate to a fully digital system, the computation complexity to determine good beamforming vectors is high. To reduce complexity, we then propose nonjoint designs of precoder and detector. For the precoder design, the transmit beamforming vectors are chosen to maximize either the signal to noise ratio or the signal to interference plus noise ratio of each user. For the detector design, the receiver beamforming vectors are selected using either an approximate ML detector or a successive cancellation detector. Through simulations, we show that our designs with low resolution phase shifters outperform the traditional methods using steering vectors as beamforming vectors with high resolution phase shifters.**

***Index Terms*—Analog beamforming, detector, hybrid beamforming, millimeter waves, phase shifters, precoder.**

I. Introduction

To MEET high data rate demand of future communications, large chunks of bandwidth must become available. Millimeter wave (mm-Wave) communications offer a promising solution, yet the main challenge is the much higher propagation path loss in the frequency range of 10–100 GHz, relative to microwave range of 2–6 GHz. Such severe path loss can be compensated by using transmit and receive beamforming through a large number of antennas. However, the complexity of digital beamforming with a large number of antennas, each with an RF chain, can be high. To reduce it, transmit and receive beamforming can be obtained by using *analog domain phase shifters* at each antenna with a single RF chain [1]−[4]. *Analog beamforming* relies on RF circuits that can be controlled to modify the phase of an incoming signal by a limited number of distinct phases [5], [6].

Selecting the best *transmit* and *receive beamforming vectors*, which align the beams along the strongest path of the channel, is a key problem in beamforming design for mm-Wave communications (see references in [7]−[9]). In [10], [30], a hierarchical search protocol for finding the strongest path of the channel was proposed. This protocol divides the search into different levels, starting from a wide beam at first level and narrowing it at each successive level. The challenge lies in the design of beamforming vectors for different beam-widths. In [10], the codebooks were designed for a phased antenna array implementing only specific four phase shifts per element and the designs in [30] considered high resolution phase shifters only. In [13], we have considered the hierarchical search for low resolution phase shifters, and proposed a general codebook design to find the beamforming vectors of different width at all levels of the hierarchical search.

Analog beamforming in its basic form supports only one stream in a point-to-point communication system and uses only one RF chain for transmitter and receiver. To support multiple streams in downlink and multiuser communication, a *hybrid beamforming* structure was proposed [4], [16], [30]. In this architecture, multiple RF chains at both transmitter and receiver are connected to all antennas, through different phase shifters each controlled by a separate beamforming vector.

For hybrid beamforming, designing multiple beamforming vectors requires the knowledge of multiple propagation paths in the RF channel, in contrast to analog beamforming which requires only the knowledge of the strongest path. Typically, at mm-Wave frequencies, a few dominant paths are sufficient to construct an accurate channel matrix model due to sparsity of propagation paths. The channel estimation and the design of transmit and receive beamforming vectors for point-to-point hybrid beamforming systems based on compressive sensing methods and orthogonal matching pursuit algorithm were discussed in [16], [30]. An efficient hierarchical codebook design for channel estimation in analog beamforming system by jointly exploiting sub-array and deactivation techniques was proposed in [17]. This method was extended to hybrid beamforming system using generalized detection probability (GDP) in [18]. In [19], the transmit and receive beamforming vectors were designed by decomposing the non-convex matrix decomposition problem into a series of convex sub-problems. The authors in [20] proposed a design of transmit and receive beamforming vectors for a finite

Manuscript received July 16, 2017; revised October 15, 2017 and November 17, 2017; accepted November 28, 2017. Date of publication December 4, 2017; date of current version April 16, 2018. This work was supported by the Australian Research Council through the Discovery Project under Grant DP160100528. The review of this paper was coordinated by Dr. E. K. S. Au. *(Corresponding author: P. Raviteja.)*

The authors are with the Department of Electrical and Computer Systems Engineering, Monash University, Clayton, VIC 3800, Australia (e-mail: Raviteja.Patchava@monash.edu; Yi.Hong@monash.edu; Emanuele.Viterbo@monash.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2017.2779407

alphabet rather than Gaussian input alphabet using an iterative gradient ascent algorithm.

Several works have focused on the mm-Wave multiuser downlink system. For example, in [23], the authors proposed a beam selection method using compressed sensing with low-cost analog beamformers. Other works for multiuser downlink were summarized in [25]–[27].

In contrast, much less work has focused on the design of beamforming vectors, in particular, transmit beamforming vectors, for mm-Wave multiuser uplink systems, where multiple users transmit simultaneously to the base station (BS). In [28], receive beamforming vectors were designed based on successive cancellation for low complexity detection at the BS. In [29], a detector that designs receive beamforming vectors at the BS using a low complexity Gram-Schmidt method was proposed. Note that, in [28], [29], only receive beamforming vectors were designed, and high resolution phase shifters were required to guarantee good performance. Recently, the authors of [32] have proposed an iterative algorithm for the design of transmit and receive beamforming vectors, assuming high resolution phase shifters and only the structure of $\mathbf{a}(\theta, N)$ (defined in (2)) for transmit and receive beamforming vectors.

In this paper, we design transmit and receive beamforming vectors for multiuser uplink system using *low resolution phase shifters only*, each with q - level phase shifts ($q = 4, 8, 16$). We assume channel state information (CSI) is available at BS. First BS computes the transmit and receive beamforming vectors and then the BS feeds back the transmit beamforming vector information to the users. The contributions of our paper are summarized below.

1) We propose a joint precoder and detector design based on the low-complexity local search algorithm for the system using low resolution phase shifters ($q = 4, 8, 16$), where both transmit and receive beamforming vectors are iteratively determined to maximize the sum-rate of the uplink system. We show by simulations that the joint design achieves similar sum-rate to the fully digital system, and better error performance than the existing scheme with high resolution phase shifters ($q = 128$) [32].
2) Although the joint design achieves excellent performance, the computation complexity of searching good beamforming vectors is high. We then propose separate designs of precoder and detector that provide good complexity-and-performance tradeoffs. In precoder designs, the transmit beamforming vectors are selected to maximize either the signal to noise ratio (SNR) or the signal to interference plus noise ratio (SINR) of each user. In detector designs, the receive beamforming vectors are chosen by using either an approximate maximum likelihood detector (AMLD) or a successive cancellation detector (SCD).
3) Through simulations, we show that our designs with low resolution phase shifters outperform the traditional methods which uses the steering vectors as beamforming vectors with high resolution phase shifters [28], [29].

The rest of the paper is organized as follows. We introduce system model in Section II, and then present the joint precoding and detector design in Section III. We present multiple independent precoding and detector designs in Section IV. The simulation results of both joint and independent designs are presented in Section V. Finally, we draw conclusions in Section VI.

We use the following notation throughout this paper: a is a scalar, $\mathbf{a}$ is a vector, and $\mathbf{A}$ is a matrix. $\mathbf{A}^{-1}$, $\mathbf{A}^T$ and $\mathbf{A}^H$ represents the inverse, transpose, and Hermitian transpose of $\mathbf{A}$ respectively. $\mathbf{I}_N$ is the N-dimensional identity matrix. The determinant of square matrix is denoted by using det $[\mathbf{A}]$. The set of $M \times N$ dimensional matrices with each entry from the complex plane is represented by $\mathbb{C}^{M\times N}$. $\mathcal{CN}(0, \sigma^2)$ represents the circularly symmetric Gaussian random variable with 0 mean and σ^2 variance. $\|\mathbf{x} - \mathbf{y}\|_0$ is used to denote the number of non-zero elements in the complex vector $(\mathbf{x} - \mathbf{y})$.

II. System Model

Consider a multiuser uplink mm-Wave system with K users and a BS, as shown in Fig. 1. We assume that each user is equipped with N_t transmit antennas over one RF chain, i.e., each user supports only one data stream to be transmitted. We also assume that all K users transmit simultaneously with perfect synchronization to the BS that contains K RF chains and N_r receive antennas. All K users and the BS adopt phase shifters for transmit and receive analog beamforming. These phase shifters change phases of analog signals by a discrete number of steps q, and the q phase shifts are uniformly distributed in $[0, 2\pi)$.

Let $\mathcal{S}_q(N)$ be the set of all possible beamforming vectors of phase shifters of a terminal with N antennas, i.e.,

$$\mathcal{S}_q(N) = \Big\{\mathbf{w} \in \mathbb{C}^{N\times 1} : w_i = e^{j\beta_i},$$
$$\beta_i \in \Big\{0, \frac{2\pi}{q}, \ldots, 2\pi\frac{q-1}{q}\Big\}\ \forall\, i = 0, 1, \ldots, N-1\Big\}. \quad (1)$$

with $\|\mathbf{w}\|^2 = N$ and $|\mathcal{S}_q(N)| = q^N$. We let $\mathbf{u}_i \in \mathbb{C}^{N_t\times 1}$ and $\mathbf{v}_j \in \mathbb{C}^{N_r\times 1}$ represent the transmit and receive beamforming vectors at the ith user and the jth RF chain of the BS, respectively, chosen from the codebooks $\mathbf{u}_i \in \mathcal{C}_t \subset \mathcal{S}_q(N_t)$, $\mathbf{v}_j \in \mathcal{C}_r \subset \mathcal{S}_q(N_r)$, $i, j = 1, \ldots, K$.

We assume uniform linear array (ULA) of antennas at all terminals (users and the BS) with $\lambda/2$ antenna spacing, where λ is the carrier wavelength. Let the antenna response vector in the angular direction θ be

$$\mathbf{a}(\theta, N) \triangleq \Big[1, e^{j\pi\cos(\theta)}, \ldots, e^{j\pi(N-1)\cos(\theta)}\Big]^T, \quad (2)$$

then the channel between the BS and the ith user, $\mathbf{H}_i \in \mathbb{C}^{N_r\times N_t}$, $i = 1, \ldots, K$, is of the form

$$\mathbf{H}_i = \frac{1}{\sqrt{L}}\sum_{l=1}^{L} \alpha_l^i \mathbf{a}(\phi_l^i, N_r)\mathbf{a}(\theta_l^i, N_t)^H, \quad (3)$$

where L is the total number of propagation paths, α_l^i is the gain of the l-th path, ϕ_l^i and θ_l^i are the corresponding angle of arrival (AoA) and angle of departure (AoD), respectively.

Let x_i be the unit power complex symbol (base-band equivalent) taken from a modulation alphabet $\mathcal{A}$ such as M-QAM, transmitted through the transmit phase shifters $\mathbf{u}_i \in \mathbb{C}^{N_t\times 1}$ of the i-th user. Then the received signal at the jth RF chain of the

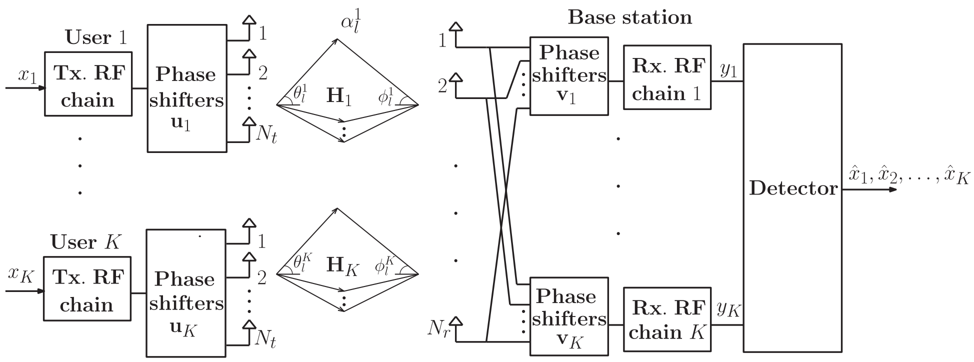

Fig. 1. Multiuser uplink millimeter wave system.

BS after using receive phase shifters $\mathbf{v}_j \in \mathbb{C}^{N_r \times 1}$, is given by

$$y_j = \sqrt{\frac{P}{N_t N_r}} \mathbf{v}_j^H \sum_{i=1}^{K} \mathbf{H}_i \mathbf{u}_i x_i + \sqrt{\frac{1}{N_r}} \mathbf{v}_j^H \mathbf{n},\ j = 1, \ldots, K, \tag{4}$$

where P is the average transmit power of each user, $\sqrt{\frac{1}{N_t}}$ and $\sqrt{\frac{1}{N_r}}$ are the normalization factors for $\mathbf{u}_i$ and $\mathbf{v}_j$, respectively, and $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is the noise vector with each entry $\sim \mathcal{CN}(0, \sigma^2)$. After the receive phase shifters, y_j, $j = 1, \ldots, K$ are passed to the detector, which is in digital domain, to estimate the transmitted signals. Different detector designs are explained in next sections. We consider SNR $= P/\sigma^2$ as the average transmit power per user to noise ratio. Here, we name $\tilde{\mathbf{h}}_i \triangleq \mathbf{H}_i \mathbf{u}_i$ as the *effective channel* for user i.

In our system, we assume the channel state information is available only at the receiving base station. Specifically, the BS first computes the transmit and receive beamforming vectors based on the proposed design, and then forwards the beamforming vectors ($N_t \log_2 q$ bits) to each user. Millimeter wave channel information can be found by using the compressive-sensing approaches as discussed in [30], [31]. These compressive-sensing approaches can be applied to analog beamforming with low resolution phase shifters, as discussed in our earlier work [13]. The algorithms proposed in [14], [15] uses AoD and channel gains to compute the beamforming vectors in the multiuser downlink system. In our paper, we assume the full channel state information, which requires AoA in addition to AoD and channel gains. Beamforming designs in multiuser uplink that uses only AoA information is very interesting and we consider it in our future work.

From (4), we clearly observe that the system performance depends on the selection of transmit and receive beamforming vectors $\mathbf{u}_i$ and $\mathbf{v}_j$, respectively, for $i, j = 1, \ldots, K$. In the following, we denote the selection of $\mathbf{u}_i$'s and $\mathbf{v}_j$'s as *precoder* and *detector design*.

III. Joint Precoder and Detector Design

In this section, we propose a joint precoder and detector[1] design for low resolution phase shifters ($q = 4, 8, 16$). The preferred transmit and receive beamforming vectors $\mathbf{u}_i$ and $\mathbf{v}_j$, $i, j \in [1, K]$ are chosen to approximately maximize the sum-rate of the uplink system by using an iterative joint search algorithm.

We first rewrite (4) as

$$\begin{aligned} \mathbf{y} &= \sqrt{P}\, \mathbf{V}^H \mathbf{H} \mathbf{U} \mathbf{x} + \mathbf{V}^H \mathbf{n} \\ &= \sqrt{P}\, \tilde{\mathbf{H}} \mathbf{x} + \mathbf{V}^H \mathbf{n}, \end{aligned} \tag{5}$$

where $\mathbf{y} \triangleq [y_1\, y_2\, \ldots\, y_K]$, $\mathbf{V} \triangleq \sqrt{\frac{1}{N_r}}\, [\mathbf{v}_1\, \mathbf{v}_2\, \ldots\, \mathbf{v}_K] \in \mathbb{C}^{N_r \times K}$, $\mathbf{H} \triangleq [\mathbf{H}_1\, \mathbf{H}_2\, \ldots\, \mathbf{H}_K] \in \mathbb{C}^{N_r \times K N_t}$, $\mathbf{x} \triangleq [x_1\, x_2\, \ldots\, x_K] \in \mathbb{C}^{K \times 1}$, $\tilde{\mathbf{H}} \triangleq \mathbf{V}^H \mathbf{H} \mathbf{U} \in \mathbb{C}^{N_r \times K}$, and

$$\mathbf{U} \triangleq \sqrt{\frac{1}{N_t}} \begin{bmatrix} \mathbf{u}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{u}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{u}_K \end{bmatrix} \in \mathbb{C}^{K N_t \times K}.$$

Let the sum-rate of the multiuser uplink system be

$$\begin{aligned} C &= \log_2 \det \left[\frac{P}{\sigma^2} \tilde{\mathbf{H}} \tilde{\mathbf{H}}^H + \mathbf{V}^H \mathbf{V} \right] \\ &= \log_2 \det \left[\frac{P}{\sigma^2} \mathbf{V}^H \mathbf{H} \mathbf{U} \mathbf{U}^H \mathbf{H}^H \mathbf{V} + \mathbf{V}^H \mathbf{V} \right] \text{ bits/s/Hz}, \end{aligned} \tag{6}$$

Then our joint design aims at finding the best $\mathbf{U}$ and $\mathbf{V}$ to maximize C, i.e.,

$$\begin{aligned} \hat{\mathbf{U}}, \hat{\mathbf{V}} &= \underset{\mathbf{U}, \mathbf{V}}{\arg\max}\ \det \left[\frac{P}{\sigma^2} \mathbf{V}^H \mathbf{H} \mathbf{U} \mathbf{U}^H \mathbf{H}^H \mathbf{V} + \mathbf{V}^H \mathbf{V} \right] \\ &= \underset{\mathbf{U}, \mathbf{V}}{\arg\max}\ f(\mathbf{U}, \mathbf{V}). \end{aligned} \tag{7}$$

where $f(\mathbf{U}, \mathbf{V})$ is named as the *metric of the joint precoder and detector design*.

This sum-rate expression is the same as the one in [24] except for the analog receive beamforming ($\mathbf{V}$) at BS which is due to the hybrid architecture used in mm-Wave systems, whereas full digital signal is used for microwave communications in [24]. After analog processing at BS, we assume optimal detection in digital stage (see Section III-B for details). Note that the optimization of the metric in (7) is different from the one considered in mm-Wave literature [25], [29], [32], where receive SINR is maximized by using linear detectors such as MMSE.

[1] We use the detector term to represent both the combiner operation at analog beamforming stage and the detection part at digital stage.

To solve (7), the brute-force search requires an exponential computation complexity of $|\mathcal{S}_q(N_t)|^K|\mathcal{S}_q(N_r)|^K = q^{K(N_t+N_r)}$. Even for a small mm-Wave system of $q = 4, K = 2, N_t = 16$, and $N_r = 16$, it requires to compute 4^{64} values. To reduce the complexity, we propose a *low complexity iterative joint precoder and detector design* algorithm, which can find a sub-optimal solution to (7).

Iterative Joint Design Algorithm: The iterative joint design algorithm starts with the randomly chosen initial values of $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$. In each iteration we sequentially update $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$ using the local search (LS) method, as discussed below. The output of the previous iteration is used as the initial solution for LS in the next iteration. The algorithm stops when the difference of C between successive iterations is arbitrarily small or when the maximum iterations are reached.

Local Search (LS) Method: LS starts with an initial solution. Then it computes the best solution to (7) in the neighborhood of the initial solution. We denote the neighborhood of $\mathbf{x} \in \mathcal{S}_q(N)$ as $\mathcal{N}_{d,N}(\mathbf{x})$, which is the set of all vectors in $\mathcal{S}_q(N)$ that differs in at most d positions from $\mathbf{x}$, i.e.,

$$\mathcal{N}_{d,N}(\mathbf{x}) \triangleq \{\mathbf{p} : \mathbf{p} \in \mathcal{S}_q(N) \text{ and } 0 < \|\mathbf{x} - \mathbf{y}\|_0 \leq d\}, \quad (8)$$

where $\|\mathbf{x} - \mathbf{y}\|_0$ denotes the number of non-zero values in $(\mathbf{x} - \mathbf{y})$ and

$$|\mathcal{N}_{d,N}(\mathbf{x})| = \sum_{i=1}^{d} \binom{N}{i} (q-1)^i.$$

The pseudocode of the proposed joint design is presented in Algorithm 1. Note that, in the pseudocode, at the t-th iteration, $\hat{\mathbf{U}}^{(t)}_{(i,\mathbf{p})}$ and $\hat{\mathbf{V}}^{(t)}_{(j,\mathbf{p})}$ are obtained by replacing $\mathbf{u}_i^{(t)}$ and $\mathbf{v}_j^{(t)}$ with the vector $\mathbf{p}$ in $\hat{\mathbf{U}}^{(t)}$ and $\hat{\mathbf{V}}^{(t)}$, respectively.

Note that a tabu search algorithm similar to this local search was proposed in [21] to find the near-optimal analog beamforming vectors for point-to-point mm-Wave system. This scheme is different from our algorithm in two aspects: i) we consider multiuser uplink system and ii) we consider the full search space $\mathcal{S}_q(N)$ for finding beamforming vectors whereas only steering vectors $(\mathbf{a}(\theta, N))$ were used in [21].

Remark 1: Please note the following points regarding the motivation of the joint design proposed in this paper. The optimal closed form solution is not known even for a full digital system without constant amplitude phase shifters constraint. An iterative solution was proposed for full digital system in [22], but its convergence has never been studied for hybrid systems. Even if an optimal iterative solution were available, approximating this solution with the hybrid beamforming matrix requires *high resolution phase shifters* and *a large number of RF chains* [16]. This is in contrast to our assumption that users have *only one RF chain* and *low resolution phase shifters*. In this case, there will be a significant performance loss, compared to the optimal solution.

Algorithm 1: The Joint Precoder and Detector Design.

Input: $\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_K$
Initialize: $\hat{\mathbf{u}}_i^{(0)}, \hat{\mathbf{v}}_j^{(0)}$, for $i, j = 1, \cdots, K$
for $t = 1$ **to** number_of_iterations **do**
  $\hat{\mathbf{U}}^{(t)} \leftarrow \hat{\mathbf{U}}^{(t-1)}$
  $\hat{\mathbf{V}}^{(t)} \leftarrow \hat{\mathbf{V}}^{(t-1)}$
  for $i = 1$ **to** K **do**
    $\mathbf{x} \leftarrow \hat{\mathbf{u}}_i^{(t)}$
    while (1) **do**
      Find $\mathbf{z} \leftarrow \underset{\mathbf{p} \in \mathcal{N}_{d,N_t}(\mathbf{x})}{\arg\max}\ f(\hat{\mathbf{U}}^{(t)}_{(i,\mathbf{p})}, \hat{\mathbf{V}}^{(t)})$
      if $f(\hat{\mathbf{U}}^{(t)}_{(i,\mathbf{z})}, \hat{\mathbf{V}}^{(t)}) > f(\hat{\mathbf{U}}^{(t)}_{(i,\mathbf{x})}, \hat{\mathbf{V}}^{(t)})$ **then**
        $\mathbf{x} \leftarrow \mathbf{z}$
      else
        break
      end if
    end while
    $\hat{\mathbf{u}}_i^{(t)} \leftarrow \mathbf{x}$,
    $\hat{\mathbf{U}}^{(t)} \leftarrow \hat{\mathbf{U}}^{(t)}_{(i,\mathbf{x})}$
  end for
  for $j = 1$ **to** K **do**
    $\mathbf{x} \leftarrow \hat{\mathbf{v}}_j^{(t)}$
    while (1) **do**
      Find $\mathbf{z} \leftarrow \underset{\mathbf{p} \in \mathcal{N}_{d,N_r}(\mathbf{x})}{\arg\max}\ f(\hat{\mathbf{U}}^{(t)}, \hat{\mathbf{V}}^{(t)}_{(j,\mathbf{p})})$
      if $f(\hat{\mathbf{U}}^{(t)}, \hat{\mathbf{V}}^{(t)}_{(j,\mathbf{z})}) > f(\hat{\mathbf{U}}^{(t)}, \hat{\mathbf{V}}^{(t)}_{(j,\mathbf{x})})$ **then**
        $\mathbf{x} \leftarrow \mathbf{z}$
      else
        break
      end if
    end while
    $\hat{\mathbf{v}}_j^{(t)} \leftarrow \mathbf{x}$
    $\hat{\mathbf{V}}^{(t)} \leftarrow \hat{\mathbf{V}}^{(t)}_{(j,\mathbf{x})}$
  end for
  if $\frac{f(\hat{\mathbf{U}}^{(t)}, \hat{\mathbf{V}}^{(t)})}{f(\hat{\mathbf{U}}^{(t-1)}, \hat{\mathbf{V}}^{(t-1)})} < 2^{\epsilon}$ **then**
    break
  end if
end for
Output: $\hat{\mathbf{u}}_i^{(t)}, \hat{\mathbf{v}}_j^{(t)}$, for $i, j = 1, \cdots, K$

A. Complexity

The complexity of the precoder design is $\mathcal{O}(Kl_p(|\mathcal{N}_{d,N_t}(\mathbf{x})|)((2K)^3 + N_r))$, where l_p represents the number of loops in LS, $|\mathcal{N}_{d,N_t}(\mathbf{x})|$ is the total number of neighbour vectors tested in each loop, and $(2K)^3 + N_r$ is the complexity taking into account of both determinant computation and the operations needed for updating $\frac{P}{\sigma^2}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H + \mathbf{V}^H\mathbf{V}$ for every neighborhood vector (only one column in $\mathbf{U}$ is updated and no updates in $\mathbf{V}$).

Similarly, the complexity of the detector design is $\mathcal{O}(Kl_d(|\mathcal{N}_{d,N_r}(\mathbf{x})|)((2K)^3 + K))$, where l_d is the number of loops in LS and $|\mathcal{N}_{d,N_r}(\mathbf{x})|$ is the total number of neighbour vectors tested in each loop.

Finally, the complexity of the joint design is $N_{\text{Iter}} \times \mathcal{C}_{\text{Iter}}$, where N_{Iter} is the total number of iterations in the joint design, and $\mathcal{C}_{\text{Iter}} = (\mathcal{O}(Kl_p(|\mathcal{N}_{d,N_t}(\mathbf{x})|)((2K)^3 + N_r)) + \mathcal{O}(Kl_d(|\mathcal{N}_{d,N_r}(\mathbf{x})|)((2K)^3 + K)))$ is the complexity of joint design per iteration.

B. Detection

The received signal vector in (5) contains the transmitted signals and the colored Gaussian noise $\mathbf{V}^H\mathbf{n}$ with covariance matrix $\Sigma = \mathbf{V}^H\mathbf{V}$. Applying maximum likelihood (ML) detection yields

$$\begin{aligned}\hat{\mathbf{x}} &= \arg\max_{\mathbf{x}\in\mathcal{A}^K} \Pr(\mathbf{y}/\tilde{\mathbf{H}}, \mathbf{x}) \\ &= \arg\min_{\mathbf{x}\in\mathcal{A}^K} (\mathbf{y} - \sqrt{P}\tilde{\mathbf{H}}\mathbf{x})^H\Sigma^{-1}(\mathbf{y} - \sqrt{P}\tilde{\mathbf{H}}\mathbf{x}) \\ &= \arg\min_{\mathbf{x}\in\mathcal{A}^K} \|\mathbf{L}^H\mathbf{y} - \sqrt{P}\mathbf{L}^H\tilde{\mathbf{H}}\mathbf{x}\|^2, \end{aligned} \tag{9}$$

where $\mathbf{L}$ is the lower triangular matrix in the Cholesky decomposition of Σ^{-1}, i.e., $\mathbf{L}\mathbf{L}^H = \Sigma^{-1}$. A sphere decoding ML can be implemented to solve (9).

C. Upper Bound on Sum-rate

The upper bound on sum-rate is computed based upon the following setting.

1) We assume fully digital systems for users and the BS, i.e., each user has N_t RF chains and the BS has N_r RF chains, and thus all N_r received signals are available at the BS.
2) We consider all users are cooperative and then the precoder matrix $\mathbf{U}_{\text{dig}}$ is a full complex-entry matrix. To maintain the total power constraint, we consider only the orthonormal vectors as the columns of $\mathbf{U}_{\text{dig}}$, i.e., $\mathbf{U}_{\text{dig}}^H\mathbf{U}_{\text{dig}} = \mathbf{I}_K$.

The received signal for the point-to-point digital system is

$$\begin{aligned}\mathbf{y}_{\text{dig}} &= \sqrt{P}\,\mathbf{H}\mathbf{U}_{\text{dig}}\mathbf{x} + \mathbf{n} \\ &= \mathbf{H}\mathbf{x}_{\text{dig}} + \mathbf{n},\end{aligned} \tag{10}$$

where $\mathbf{x}_{\text{dig}} = \mathbf{U}_{\text{dig}}\mathbf{x}$ and $\|\mathbf{x}\|^2 = \|\mathbf{x}_{\text{dig}}\|^2$. The achievable rate of the system in (10) is given by

$$C_{\text{dig}} = \log_2 \det\left[\frac{P}{\sigma^2}\mathbf{H}\mathbf{H}^H + \mathbf{I}_{N_r}\right] \text{ bits/s/Hz.} \tag{11}$$

By using singular value decomposition (SVD) of $\mathbf{H}$, the rate in (11) can be written as [33],

$$C \leq C_{\text{dig}} = \sum_{i=1}^{K} \log_2\left(1 + \frac{\lambda_i^2 P}{\sigma^2}\right) \text{ bits/s/Hz,} \tag{12}$$

where λ_i, for $i = 1, \ldots, K$ are the K largest singular values of $\mathbf{H}$.

Through the simulation results in Section V-A, we can observe that the sum-rate of the proposed joint precoder and detector design is very close to the upper bound. Although the joint design performs well, it requires high complexity. To reduce the complexity, we propose separate designs of precoder and detector in the next section.

IV. Separate Precoder and Detector Design

In this section, we present the two precoder designs based on SNR and SINR maximization, followed by the two detector designs.

A. The Precoder Design

We propose two different precoder designs for limited resolution phase shifters ($q = 4, 8, 16$).

1) Conventional Precoder (CP): For user i with its known CSI, the conventional precoder selects $\mathbf{u}_i$ that maximizes the total effective channel power, i.e.,

$$\mathbf{u}_i = \arg\max_{\mathbf{x}\in\mathcal{S}_q(N_t)} \|\mathbf{H}_i\mathbf{x}\|^2, \text{for } i = 1, \ldots, K. \tag{13}$$

This design requires a very high processing time as $|\mathcal{S}_q(N_t)| = q^{N_t}$. For example, in the case of $q = 16$ and $N_t = 16$, we need to check for 16^{16} values, which is impractical. To reduce the complexity, we adopt LS method to find an approximate solution to (13). In LS, we consider $\mathcal{E}(\mathbf{x}) \triangleq \|\mathbf{H}_i\mathbf{x}\|^2$ as the *metric of the algorithm*. The complexity of this algorithm is $\mathcal{O}(Kl_p|\mathcal{N}_{d,N_t}(\mathbf{x})|)$, much less than joint design in Section III.

However, the performance of CP can be poor due to high correlation between the effective channels of different users. Consequently, the BS fails to distinguish different users, since the precoder design is based only on the respective channel of each user. To reduce such correlations by taking advantage of other users' CSIs, we propose below a successive estimation precoder (SEP).

2) Successive Estimation Precoder (SEP): We let $i_1, i_2, \ldots, i_K \in \{1, \ldots, k, \ldots, K\}$ be the *users order*, for which each precoding vector is successively computed to maximize its SINR. Here, the SINR of user i_k is defined as the ratio of the user's total effective channel power and the interference from users $i_1, \ldots, i_{k-1}$ together with the noise.

3) Matched-Filter (MF) Based SINR: For user i_k, based on the MF detector weights (i.e., $(\mathbf{H}_{i_k}\mathbf{x})^H/\|\mathbf{H}_{i_k}\mathbf{x}\|$) for its SINR computation, the SEP selects $\mathbf{u}_{i_k}$ such as

$$\begin{aligned}\mathbf{u}_{i_k} &= \arg\max_{\mathbf{x}\in\mathcal{S}_q(N_t)} \frac{\|\mathbf{H}_{i_k}\mathbf{x}\|^2}{\sum_{j=1}^{k-1}\left|\frac{(\mathbf{H}_{i_k}\mathbf{x})^H}{\|\mathbf{H}_{i_k}\mathbf{x}\|}(\mathbf{H}_{i_j}\mathbf{u}_{i_j})\right|^2 + N_t\sigma^2}, \\ &= \arg\max_{\mathbf{x}\in\mathcal{S}_q(N_t)} \mathcal{Z}_{i_k}^k(\mathbf{x}), \text{ for } k = 1, \ldots, K.\end{aligned} \tag{14}$$

4) MMSE Based SINR: For user i_k, assuming the interference only from users $i_1, i_2, \ldots, i_{k-1}$, we apply the MMSE detector weights $\mathbf{w}_{i_k}^k$ to (14) to select

$$\begin{aligned}\mathbf{u}_{i_k} &= \arg\max_{\mathbf{x}\in\mathcal{S}_q(N_t)} \frac{\mathbf{w}_{i_k}^k\mathbf{H}_{i_k}\mathbf{x}}{\sum_{j=1}^{k-1}\left|\mathbf{w}_{i_k}^k\mathbf{H}_{i_j}\mathbf{u}_{i_j}\right|^2 + N_t\sigma^2}, \\ &= \arg\max_{\mathbf{x}\in\mathcal{S}_q(N_t)} \mathcal{M}_{i_k}^k(\mathbf{x}), \text{ for } k = 1, \ldots, K.\end{aligned} \tag{15}$$

By letting $\mathbf{H}_{\text{eff}} \triangleq [\mathbf{H}_{i_k}\mathbf{x}\ \mathbf{H}_{i_1}\mathbf{u}_{i_1}\ \ldots\ \mathbf{H}_{i_{k-1}}\mathbf{u}_{i_{k-1}}] \in \mathbb{C}^{N_r\times k}$, the MMSE weight matrix $\mathbf{W}_{\text{mmse}}^k \in \mathbb{C}^{k\times N_r}$ can be written as

$$\mathbf{W}_{\text{mmse}}^k \triangleq (\mathbf{H}_{\text{eff}}^H\mathbf{H}_{\text{eff}} + N_t\sigma^2\mathbf{I})^{-1}\mathbf{H}_{\text{eff}}^H. \tag{16}$$

Therefore, $\mathbf{w}_{i_k}^k \in \mathbb{C}^{1\times N_r}$ is the normalized version of the first row of $\mathbf{W}_{\text{mmse}}^k$, corresponding to user i_k.

Finding the user order for precoder design: The user order can be randomly chosen. However, to improve the performance, at stage k, we choose the i_k-th user as the one

Algorithm 2: The Successive Estimation Precoder.

```
1: Input: H_1, H_2, ..., H_K
2: for k = 1 to K do
3:     for l ∈ U_k do
4:         p_l ← LS(Z_l^k(x))
5:     end for
6:     i_k ← arg min_{l∈U_k} Z_l^k(p_l)
7:     u_{i_k} ← p_{i_k}
8: end for
9: Output: u_1, u_2, ..., u_K
```

which has the least maximum SINR from the user index set $\mathcal{U}_k = \big\{\{1,2,\ldots,K\} - \{i_1,i_2,\ldots,i_{k-1}\}\big\}$. We name this order as *minimum order* (see more detailed discussions about alternative user ordering in Section V).

At the initial stage, we choose the i_1-th user which has the least maximum effective channel power, since we disregard the interference from others. This is because, as the number of stages increases, the number of interference terms increases, which puts more constraints on the selection of precoding vector, yielding a further reduction in SNR of that user. With the user ordering, SEP design requires $\frac{K(K+1)}{2}$ of computations in (14). Similar idea applies to the MMSE based scheme. The corresponding pseudocode is given in Algorithm 2.

To reduce the exponential computation complexity of (14), we adopt the LS method in Algorithm 2 to provide sub-optimal solutions to (14) prior to deciding the i_k-th user order. To further reduce the complexity of LS method, for user i_k, we use the first stage solution of SEP as the initial solution in the next stage and then update it at each stage.

Overall Algorithm 2 works as follows: In the first stage, the precoding vectors of all the users are computed by maximizing their corresponding SINR using LS method. Then the user with the least SINR is selected and its precoding vector is finalized. In the next stage, the precoding vectors of the users except the selected user in the first stage $\mathcal{U}_2 = \big\{\{1,2,\ldots,K\} - \{i_1\}\big\}$ are computed by maximizing the modified SINR and the user with the least SINR is selected and its precoding vector is finalized. This process continues until precoding vectors of all users are finalized. The complexity order of Algorithm 2 is $\mathcal{O}(K^2 l_p |\mathcal{N}_{d,N_t}(\mathbf{x})|)$.

B. The Detector Design

In this subsection, we propose two different detectors: one provides an approximate solution to ML detection and the other is based on successive cancellation. In both designs, we assume that the $\mathbf{u}_i$'s are already made available using computations in Section IV-A.

1) Approximate ML Detector (AMLD): Given the known received signal $\mathbf{s} \in \mathbb{C}^{N_r \times 1}$ (the received signal at the BS before phase shifters) and assuming that the BS adopts N_r RF chains (the optimal case), the ML detection is given by

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathcal{A}^K}{\arg\min} \|\mathbf{s} - \tilde{\mathbf{H}}\mathbf{x}\|^2, \tag{17}$$

where $\tilde{\mathbf{H}} \triangleq [\tilde{\mathbf{h}}_1, \tilde{\mathbf{h}}_2, \ldots, \tilde{\mathbf{h}}_K] \in \mathbb{C}^{N_r \times K}$, $\hat{\mathbf{x}} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_K]^T$, and $\hat{x}_i$ is the decoded symbol corresponding to user i. Considering

$$\tilde{\mathbf{H}} = \mathbf{QR} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 \end{bmatrix} \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{bmatrix}, \tag{18}$$

where $\mathbf{Q} \in \mathbb{C}^{N_r \times N_r}$ is a unitary matrix and $\mathbf{R} \in \mathbb{C}^{N_r \times K}$ is an upper triangular matrix with the last $N_r - K$ rows as completely zeros, the ML detection in (17) can be rewritten as

$$\begin{aligned} \hat{\mathbf{x}} &= \underset{\mathbf{x}\in\mathcal{A}^K}{\arg\min} \|\mathbf{Q}^H\mathbf{s} - \mathbf{R}\mathbf{x}\|^2, \\ &= \underset{\mathbf{x}\in\mathcal{A}^K}{\arg\min} \|\mathbf{Q}_1^H\mathbf{s} - \mathbf{R}_1\mathbf{x}\|^2, \end{aligned} \tag{19}$$

where $\mathbf{Q}_1 \in \mathbb{C}^{N_r \times K}$ is the matrix with first K columns of $\mathbf{Q}$ and $\mathbf{R}_1 \in \mathbb{C}^{K \times K}$ is the matrix with first K rows of $\mathbf{R}$. From (19), we observe that ML detection can also be implemented by using $\mathbf{Q}_1^H\mathbf{s} \in \mathbb{C}^{K\times 1}$ instead of $\mathbf{s} \in \mathbb{C}^{N_r \times 1}$.

Therefore, we apply ML detection in our hybrid beamforming at the BS by $i)$ letting the $\mathbf{v}_j$'s equal to the columns in $\mathbf{Q}_1$ to obtain $\mathbf{Q}_1^H\mathbf{s}$ in analog domain, and $ii)$ applying (19) on $\mathbf{Q}_1^H\mathbf{s}$ in digital domain. Unfortunately, the elements of $\mathbf{v}_j$'s have a constant amplitude constraint and cannot be made equal to those of $\mathbf{Q}_1$ which have different amplitudes and phases. To solve this problem, we propose a solution that is based on the following Lemma derived in [11], [12].

Lemma 1. Every element of $\mathbf{Q}_1$ can be written as a sum of two unit amplitude values. That is, (i,j)-th element of $\mathbf{Q}_1$, $e_{i,j}$, can be written as

$$e_{i,j} = e^{j(\cos^{-1}(a_{i,j}/2)+\theta_{i,j})} + e^{j(-\cos^{-1}(a_{i,j}/2)+\theta_{i,j})}, \tag{20}$$

where, $a_{i,j}$ and $\theta_{i,j}$ are the amplitude and phase of $e_{i,j}$ respectively. ∎

Since a large array of antennas is used in mm-Waves to compensate path-loss, in most of the cases, the value of $a_{i,j}$ is close to 0 and the difference of the angles in (20), $2\cos^{-1}(a_{i,j}/2)$, is close to 180^0, i.e., one angle in $[0,\pi)$ and the other in $[\pi, 2\pi)$. Therefore, $[\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_K] = \mathbf{Q}_1$ is implemented in the analog domain with $2N_rK$ high resolution discrete phase shifters, of which N_rK covers the range $[0,\pi)$ and the other N_rK covers the remaining range.

We name it as *approximate ML detection (AMLD)*, since discrete phase shifters are used rather than continuous ones. In Section V, we present the BER performance with different resolutions of phase shifters (16, 32, 64) and show that with high resolution phases shifters, we can approach the optimal performance. Although the performance of AMLD is close to optimal, it requires twice amount (i.e., $2N_rK$) of phase shifters of high resolution, when compared to a regular hybrid beamforming structure. To tackle this problem, in the following subsection, we present a detector which requires much lesser phase shifters of *low resolution* with trade-offs in performance.

2) Successive Cancellation Detector (SCD): Similar to the SEP in Section IV-A2, this detector is also implemented in K stages. Let $j_1, \ldots, j_k, \ldots, j_K \in \{1,2,\ldots,K\}$ be the *users order* at which the corresponding decoding vectors $\{\mathbf{v}_{j_k}\}_{k=1}^K$

Algorithm 3: The Successive Cancellation Detector.

Input: $\tilde{\mathbf{h}}_1, \tilde{\mathbf{h}}_2, \cdots, \tilde{\mathbf{h}}_K$
for $k = 1$ **to** K **do**
  for $l \in \mathcal{V}_k$ **do**
    $\mathbf{p}_l \leftarrow \text{LS}(\mathcal{D}_l^k(\mathbf{x}))$
  end for
  $j_k \leftarrow \arg\max_{l \in \mathcal{V}_k} \mathcal{D}_l^k(\mathbf{p}_l)$
  $\mathbf{v}_{j_k} \leftarrow \mathbf{p}_{j_k}$
end for
Output: $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_K$

are successively computed. At stage k, we select the detection vector for user j_k as

$$\begin{aligned} \mathbf{v}_{j_k} &= \arg\max_{\mathbf{x} \in \mathcal{S}_q(N_r)} \frac{|\mathbf{x}^H \tilde{\mathbf{h}}_{j_k}|^2}{\sum_{i=k+1}^{K} \left|\mathbf{x}^H \tilde{\mathbf{h}}_{j_i}\right|^2 + N_r \sigma^2}, \\ &= \arg\max_{\mathbf{x} \in \mathcal{S}_q(N_r)} \mathcal{D}_{j_k}^k(\mathbf{x}), \text{ for } k = 1, \ldots, K. \end{aligned} \tag{21}$$

Note that the interference term in (21) only takes into account those users for which decoding vectors are not computed. Similar to (13), the computation of (21) requires q^{N_r} comparisons, which is impractical for large values of N_r (32, 64). We thus use the local search to find the approximate solution to (21) with metric $\mathcal{D}_{j_k}^k(\mathbf{x})$.

Finding user order for detector design: At stage k, we choose user j_k as the one with the highest maximum SINR, i.e., $\max_{l \in \mathcal{V}_k}(\max_{\mathbf{x} \in \mathcal{S}_q(N_r)} \mathcal{D}_l^k(\mathbf{x}))$, from the user set $\mathcal{V}_k = \{\{1, 2, \ldots, K\} - \{j_1, j_2, \ldots, j_{k-1}\}\}$. We name this order as *maximum order*. As the stage increases, fewer interference terms are involved in (21) and also the interference from high SINR users $j_1, \ldots, j_{k-1}$ is removed, leading to improved performance for users with low SINR.

The corresponding pseudocode is given in Algorithm 3. The LS method is used to find sub-optimal solution to $\max_{\mathbf{x} \in \mathcal{S}_q(N_r)} \mathcal{D}_l^k(\mathbf{x})$ for a given $l \in \mathcal{V}_k$. The complexity order of this algorithm is $\mathcal{O}(K^2 l_d |\mathcal{N}_{d,N_r}(\mathbf{x})|)$.

Detection Rule: The detector follows the same order as above to detect the transmitted signals $x_{j_1}, \ldots, x_{j_k}, \ldots, x_{j_K}$. In particular, the detection rule for signal x_{j_k} is

$$\begin{aligned} \hat{x}_{j_k} &= \arg\min_{z \in \mathcal{A}} \left| \mathbf{v}_{j_k}^H \left(\mathbf{s} - \sum_{i=1}^{k-1} \tilde{\mathbf{h}}_{j_i} \hat{x}_{j_i} \right) - \mathbf{v}_{j_k}^H \tilde{\mathbf{h}}_{j_k} z \right|^2, \\ &= \arg\min_{z \in \mathcal{A}} \left| y_{j_k} - \sum_{i=1}^{k-1} \mathbf{v}_{j_k}^H \tilde{\mathbf{h}}_{j_i} \hat{x}_{j_i} - \mathbf{v}_{j_k}^H \tilde{\mathbf{h}}_{j_k} z \right|^2, \end{aligned} \tag{22}$$

where $\mathbf{s} - \sum_{i=1}^{k-1} \tilde{\mathbf{h}}_{j_i} \hat{x}_{j_i}$ denotes the received signal after canceling the interference from users $j_1, \ldots, j_{k-1}$. Here, we can see that the interference from users $j_{k+1}, \ldots, j_K$ in $\mathbf{v}_{j_k}^H$ is neglected, simply because we design $\mathbf{v}_{j_k}^H$ based on reducing the interference from these set of users (see (21)). Different from [28], our detector uses LS to select $\mathbf{v}_i$ for low-resolution phase shifters, whereas in [28], the detector was designed for high resolution phase shifters by considering only the detection vectors ($\mathbf{v}_i$) of structure $\mathbf{a}(\phi, N_r)$, i.e., one parameter ϕ decides the entire vector $\mathbf{v}_i$.

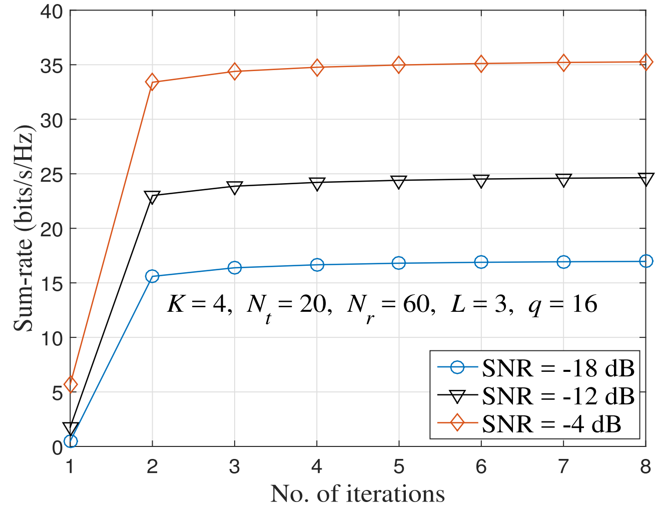

Fig. 2. The sum-rate of the proposed joint design as a function of number of iterations at different SNR's with $N_t = 20, N_r = 60, K = 4, q = 16$, and $L = 3$.

V. Simulation Results And Discussion

In this section, we first present the sum-rate of the proposed joint precoder and detector design and compare with the upper bound achieved by the point-to-point digital system. We then compare the bit error rate (BER) performance using our proposed joint design as well as separate precoder and detector designs. We also compare our system performance with the performance of other existing schemes in [28], [29], and [32] as well as a fully digital system with ML detection.

A. Joint Precoder and Detector Design

In simulations, we adopt the following setting: $K = 4$, $N_t = 20$, $N_r = 60$, and QPSK signalling for transmission. The channel between the BS and each user has $L = 3$ propagation paths, where one path is line-of-sight (LoS) and the other two are non-line-of-sight (NLoS) paths. For LoS path, we assume the path gain as $\alpha_1^i = 1$, for $i = 1, \ldots, K$, and for NLoS paths, the path gain as $(\alpha_l^i) \sim \mathcal{CN}(0, \frac{1}{\sqrt{L}})$, where $l = 2, \ldots, L$ and $i = 1, \ldots, K$ [28]. The AoD (θ_l^i) is uniformly distributed in $[0, 2\pi)$ and the AoA (ϕ_l^i) is uniformly distributed in $[-\pi/3, +\pi/3)$ due to sectorization at BS. For LS, we adopt $d = 1$ in (8) to list the neighborhood of $\mathbf{x}$.

In Fig. 2, we illustrate the variation of the sum-rate of the proposed joint design with the number of iterations at different SNRs for low resolution phase shifter $q = 16$. We observe that the proposed joint design approaches the maximum sum-rate in only 3 iterations. In this simulation, we found the average number of loops in LS are $l_p = 30$ for precoder design and $l_d = 90$ for detector design, respectively.

Figs. 3 and 4 show the variation of sum-rate as a function of number of iterations and neighborhood size (d) with

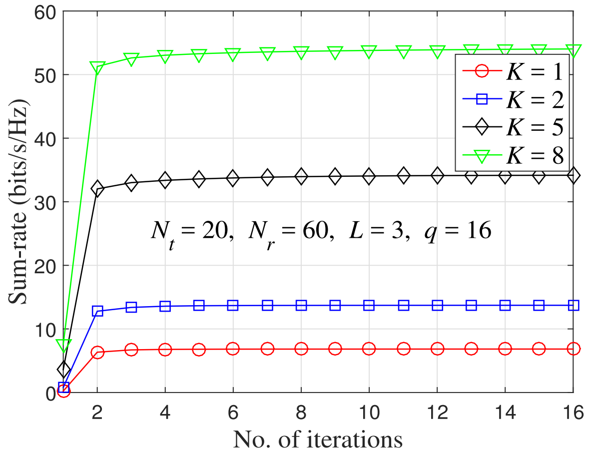

Fig. 3. The sum-rate of the proposed joint design as a function of number of iterations for different number of users with $N_t = 20, N_r = 60, P/\sigma^2 = -10$ dB, and $L = 3$.

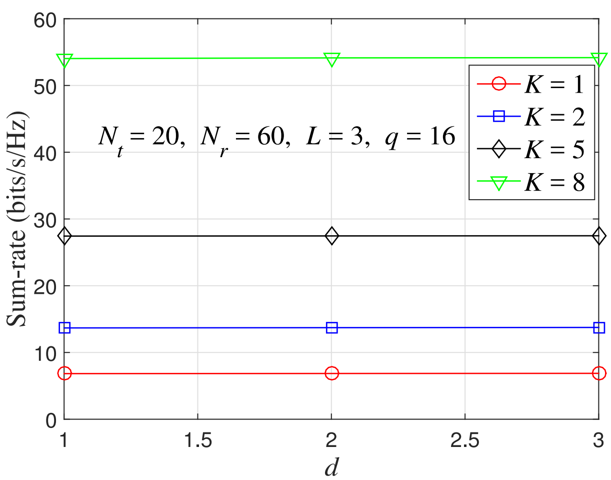

Fig. 4. The sum-rate of the proposed joint design as a function of neighborhood size d for different number of users (K) with $N_t = 20, N_r = 60, P/\sigma^2 = -10$ dB, and $L = 3$.

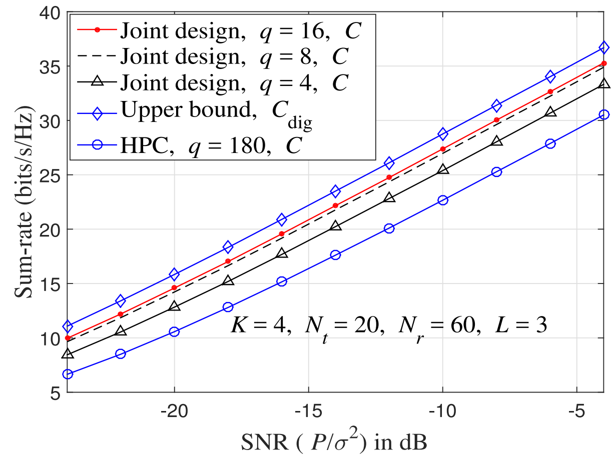

Fig. 5. The sum-rate of the proposed joint design at different SNR's with $N_t = 20, N_r = 60, K = 4$, and $L = 3$.

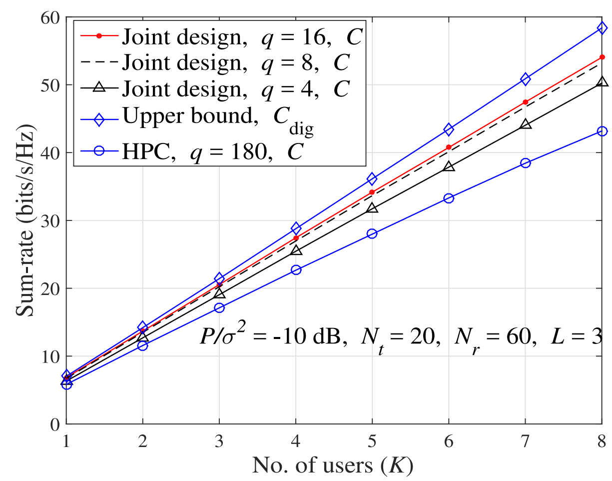

Fig. 6. The sum-rate of the proposed joint design for different number of users (K) with $N_t = 20, N_r = 60, P/\sigma^2 = -10$ dB, and $L = 3$.

different number of users ($K = 1, 2, 5, 8$) with $N_t = 20$, $N_r = 60$, $P/\sigma^2 = -10$ dB, and $L = 3$, respectively. We can see that the joint design is converging in 3 iterations for different number of users. This behavior may be heuristically explained by the fact that the neighborhood size is large enough to avoid the local traps in the search. We also observe that the increase in value of d does not effect the performance as the size of neighborhood is large even with $d = 1$, i.e., $N_t(q-1)$ or $N_r(q-1)$. Finally, convergence analysis of the local search algorithm is an interesting open problem that has not been solved yet in the literature.

Figs. 5 and 6 illustrate the sum-rate of the proposed joint design at different SNRs and for different number of users, respectively, with low resolution phase shifters $q = 4, 8, 16$. In both figures, we also compare the sum-rate of our design with the corresponding upper bound $C_{\rm dig}$ and those of the iterative hybrid precoder and combiner design (HPC) [32]. We observe that the sum-rate of our design outperforms HPC by approximately 5 bits/s/Hz thanks to fully exploitation of all possible combinations for precoders and detectors ($\mathcal{S}_q(N_t)$ and $\mathcal{S}_q(N_r)$) rather than constraining to the form of $\mathbf{a}(\theta, N)$ as in [32]. We also see that our design with $q = 8, 16$ is the closest to the upper bound among all, though the joint design has overall high complexity, as discussed in Section III A.

B. Separate Precoder and Detector Design

In the simulations, we consider the settings presented in the subsection V-A. For BER computations, we consider the SNR as E_b/σ^2.

Fig. 7 shows the BER of the joint precoder and detector design with low resolution phase shifters of $q = 4, 8, 16$. We observe that the performance improves as q increases. We also see that the joint design with $q = 8$ and $q = 16$ significantly outperforms that with $q = 4$.

Fig. 8 compares the BER performance of CP and SEP using MF and MMSE weights for SINR computations, denoted by SEP-MF and SEP-MMSE. We observe that: i) SEP-MMSE outperforms CP and SEP-MF (more than 4 dB gain at 10^{-4} BER), ii) performance improves as q increases, and iii) all the schemes exhibit an error floor at high SNRs due to the

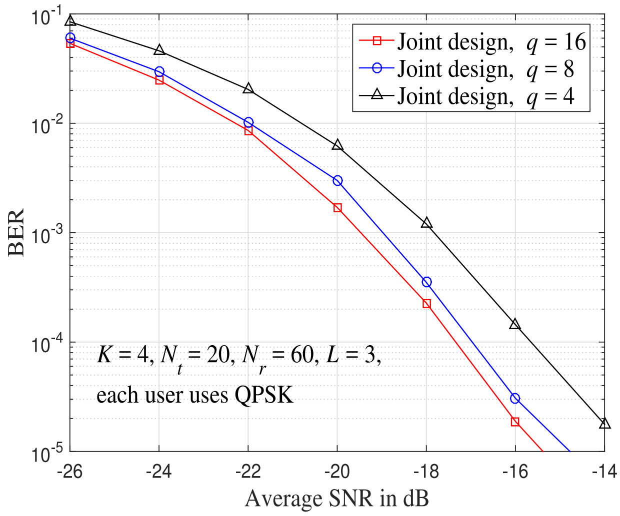

Fig. 7. The BER performance of CP, SEP-MF, and SEP-MMSE precoders with SCD detector for different values of q with $N_t = 20, N_r = 60, K = 4$, and $L = 3$.

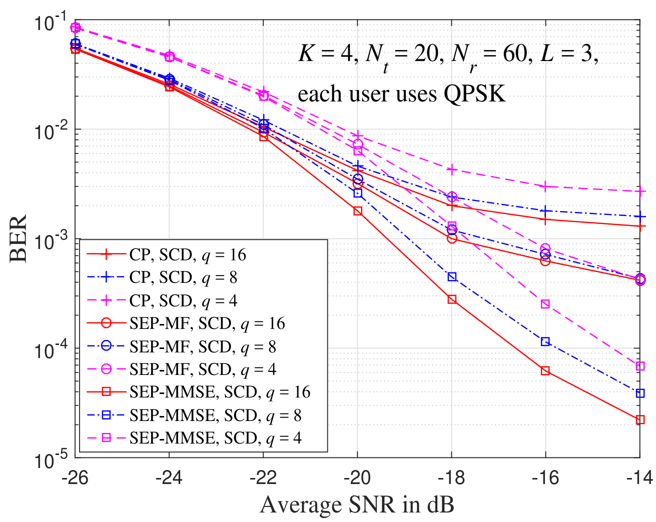

Fig. 8. The BER performance of CP, SEP-MF, and SEP-MMSE precoders with SCD detector for different values of q with $N_t = 20, N_r = 60, K = 4$, and $L = 3$.

residual correlations. This is due to the sparsity of mm-wave channels, where there are more chances of having correlations between the channels of different users. If the users' channels are highly correlated, the base station fails to recognize different users and cannot detect the signal, even at high SNR with any precoding and detection scheme. This is why there is flooring effect in all plots. Note that this flooring effect occurs at different BERs for different detection schemes, mainly depending on their interference cancellation capability.

The performance of SEP-MMSE precoder using different user orders is illustrated in Fig. 9. The SCD proposed in Section IV-B2 is used at the BS. Three different precoder users orders are considered: $i)$ maximum order: the user with the largest maximum SINR at every stage (considered $\arg\max$ at line number 6 in Algorithm 2), $ii)$ fixed order: the user order of $1, 2, \cdots, K$, $iii)$ minimum order: the user with the least

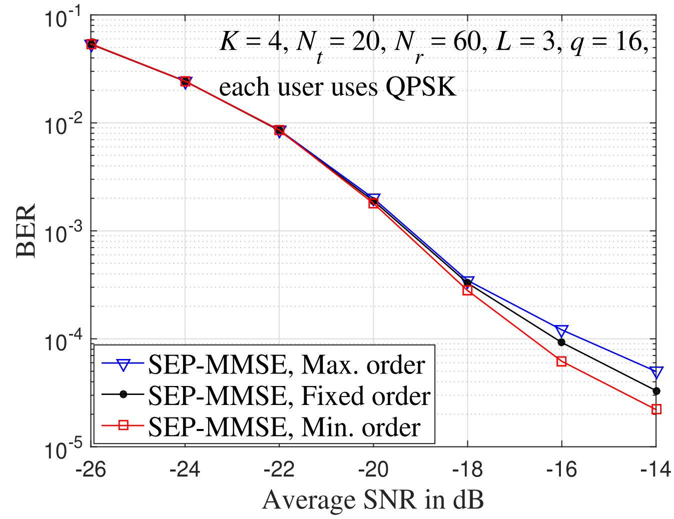

Fig. 9. The BER performance of SEP-MMSE precoder with SCD detector for different user orders with $N_t = 20, N_r = 60, K = 4$, q = 16 and $L = 3$.

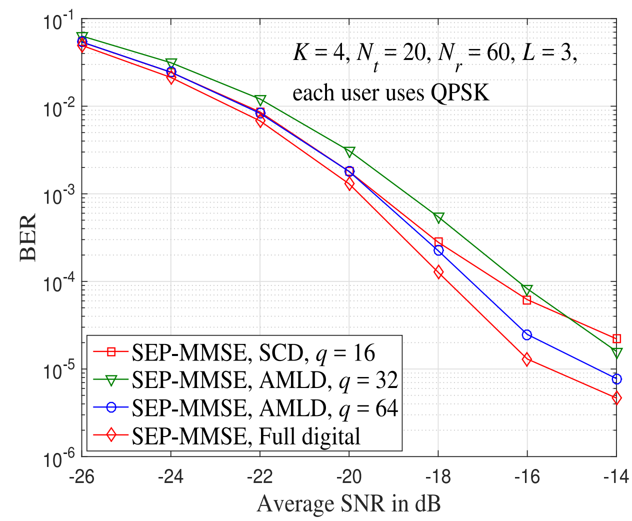

Fig. 10. The BER performance of SEP-MMSE precoder with SCD and AMLD detectors for different values of q with $N_t = 20, N_r = 60, K = 4$, and $L = 3$.

maximum SINR at every stage, as in Algorithm 2. We can observe that, at high SNRs, the precoder with the minimum order performs the best.

In Fig. 10, we compare the performance of a fully digital system using ML detection and our system using SEP-MMSE precoder with SCD and AMLD at the receiver, respectively. The AMLD uses 480 ($60 \times 4 \times 2$) phase shifters of resolutions $q = 16, 32, 64$ with 240 covering the range $[0, \pi)$ and the other 240 covering $[\pi, 2\pi)$. We can see that the BER of AMLD with $q = 64$ is closest to the fully digital performance, when compared to the others, while the SCD achieves reasonably good performance with 240 low resolution phase shifters.

In Fig. 11, we compare the performance of various systems: $i)$ using a SEP-MMSE precoder at transmitter but different detection schemes at receiver: a fully digital one with ML detection, our system using SCD, the systems using the ordered SIC method in [28], and Gram-Schmidt (HBF-GS) method in [29]

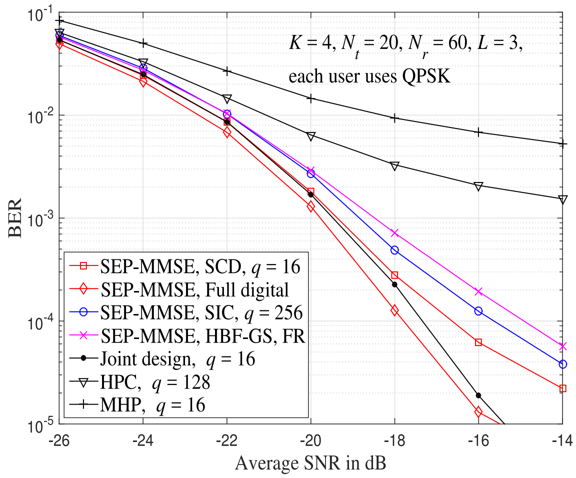

Fig. 11. The BER performance of SEP-MMSE precoder with SCD detector, SIC, HBF-GS, MHP, and HPC for $N_t = 20$, $N_r = 60$, $K = 4$, and $L = 3$.

for receive beamforming design, ii) two-stage multiuser hybrid precoder (MHP) in [27] , iii) the HPC design in [32], and iv) joint precoder and detector design. Here, we assume that ordered SIC uses 240 phase shifters with $q = 256$ resolution, HBF-SC uses 240 phase shifters of full resolution (FR), i.e., it covers all continuous range of angles, MHP uses $q = 16$ resolution phase shifters, and HPC uses $q = 128$ resolution phase shifters.

Complexity Comparison: The schemes in [28], [29] only design the detector assuming the precoding vectors are already available. The complexity order of both designs is $\mathcal{O}(N_r K^2)$, which is similar to our SCD when $K << N_r$. The HPC design in [32] is a joint precoder and detector design with complexity order $\mathcal{O}(N_r N_t K^2)$, which is also similar to our joint design. Through simulations, we observe the similar average CPU running times for all these methods.

From Fig. 11, we see that the joint design performs closely to the fully digital one, and better than all the other designs. We also observe various detector designs together with our SEP-MMSE precoder have better performance than MHP [27] and HPC [32], where an error floor occurs at 10^{-3} BER. This is due to the fact that MHP selects the analog beamforming vectors of a user based on its own channel and the interference between users is handled only by the digital stage at BS. Further, hybrid precoder and combiner (HPC) design in [32] aims at maximizing the multiuser sum-rate, when a linear MMSE linear detector is adopted at the receiver. The BER of HPC is mainly limited by this linear MMSE detector, while other schemes are non-linear and able to cancel the interference more effectively.

Moreover we see that our SCD detector with low resolution phase shifters ($q = 16$) outperforms the SIC system [28] and HBF-GS [29] with FR phase shifters by 1 dB and 2 dB, respectively, at BER of 10^{-4}. This performance gain is because of the exploration of more combinations for beamforming vectors ($\mathcal{S}_q(N)$) rather than constrained to a particular form of $\mathbf{a}(\theta, N)$.

VI. Conclusion

In this paper, we have considered a mm-Wave multiuser uplink system for low resolution phase shifters ($q = 4, 8, 16$). We have proposed a joint precoder and detector design to maximize the sum-rate of the uplink system. Although the joint design approaches the sum-rate of a fully digital system, the computation complexity to determine the good beamforming vectors is high. Hence, we have separately designed precoders and detectors, to provide a tradeoff between complexity and performance. For the precoder designs, the preferred transmit beamforming vectors are chosen to maximize either SNR or SINR of each user, using a low complexity LS method. For detector designs, the receiver beamforming vectors are selected using AMLD and SCD, respectively. Although AMLD has similar performance to the fully digital system, it requires to double the number of phase shifters and a higher phase resolution, when compared to a regular hybrid beamforming receiver. In contrast, the SCD uses only low resolution phase shifters without doubling their number. We have shown by simulations that both joint design and separate designs with low resolution phase shifters outperform the traditional methods using steering vectors as beamforming vectors with high resolution phase shifters.

References

[1] T. S. Rappaport *et al.*, “Millimeter wave mobile communications for 5G cellular: It will work!,” *IEEE Access*, vol. 1, pp. 335–349, May 2013.

[2] S. Rangan, T. Rappaport, and E. Erkip, “Millimeter-wave cellular wireless networks: Potentials and challenges,” *Proc. IEEE*, vol. 102, no. 3, pp. 366–385, Mar. 2014.

[3] F. Gutierrez, K. Parrish, and T. S. Rappaport, “On-chip integrated antenna structures in CMOS for 60 GHz WPAN systems,” *IEEE J. Sel. Areas Commun.*, vol. 27, no. 8, pp. 1367–1378, Oct. 2009.

[4] S. Kutty and D. Sen, “Beamforming for millimeter wave communications: An inclusive survey,” *IEEE Commun. Surveys Tuts.*, vol. 18, no. 2, pp. 949–973, May 2016.

[5] J. Roderick, H. Krishnaswamy, K. Newton, and H. Hashemi, “Silicon-based ultra-wideband beam-forming,” *IEEE J. Solid-State Circuits*, vol. 41, no. 8, pp. 1726–1739, Aug. 2006.

[6] Z. Yonghong, F. Zhenghe, and F. Yong, “Ka-band 4-bit phase shifter with low phase deviation,” in *Proc. Int. Conf. Microw. Millimeter Wave Technol.*, Aug. 2004, pp. 382–385.

[7] P. Xia, S. K. Yong, J. Oh, and C. Ngo, “Multi-stage iterative antenna training for millimeter wave communications,” in *Proc. IEEE Global Telecommun. Conf.*, New Orleans, LA, USA, Dec. 2008, pp. 1–6.

[8] Y. Peng, Y. Li, and P. Wang, “An enhanced channel estimation method for millimeter wave systems with massive antenna arrays,” *IEEE Commun. Lett.*, vol. 19, no. 9, pp. 1592–1595, Sep. 2015.

[9] Z. Xiao, L. Bai, and J. Choi, “Iterative joint beamforming training with constant-amplitude phased arrays in millimeter-wave communications,” *IEEE Commun. Lett.*, vol. 18, no. 5, pp. 829–832, May 2014.

[10] J. Wang *et al.*, “Beam codebook based beamforming protocol for multi-Gbps millimeter-wave WPAN systems,” *IEEE J. Sel. Areas Commun.*, vol. 27, no. 8, pp. 1390–1399, Oct. 2009.

[11] F. Sohrabi and W. Yu, “Hybrid digital and analog beamforming design for large-scale antenna arrays,” *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 3, pp. 501–513, Apr. 2016.

[12] T. E. Bogale, L. B. Le, A. Haghighat, and L. Vadndendorpe, “On the number of RF chains and phase shifters, and scheduling design with hybrid analog-digital beamforming,” *IEEE Trans. Wireless Commun.*, vol. 15, no. 5, pp. 3311–3326, May 2016.

[13] P. Raviteja, Yi Hong, and E. Viterbo, “Analog beamforming with low resolution phase shifters,” *IEEE Wireless Commun. Lett.*, vol. 6, no. 4, Aug. 2017.

[14] R. Rajashekar and L. Hanzo, “Iterative matrix decomposition aided block diagonalization for mm-Wave multiuser MIMO systems,” *IEEE Trans. Wireless Commun.*, vol. 16, no. 3, pp. 1372–1384, Mar. 2017.

[15] R. Rajashekar and L. Hanzo, “User selection algorithms for block diagonalization aided multiuser downlink mm-Wave communication,” *IEEE Access*, vol. 5, pp. 5760–5772, Mar. 2017.

[16] O. El Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. Heath, "Spatially sparse precoding in millimeter wave MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499–1513, Mar. 2014.

[17] Z. Xiao, T. He, P. Xia, and X.-G. Xia, "Hierarchical codebook design for beamforming training in millimeter-wave communication," *IEEE Trans. Wireless Commun.*, vol. 15, no. 5, pp. 3380–3392, May 2016.

[18] Z. Xiao, P. Xia, and X.-G. Xia, "Codebook design for millimeter-wave channel estimation with hybrid precoding structure," *IEEE Trans. Wireless Commun.*, vol. 16, no. 1, pp. 141–153, Jan. 2017.

[19] W. Ni, X. Dong, and W. S. Lu, "Near-optimal hybrid processing for massive MIMO systems via matrix decomposition," *IEEE Trans. Signal Process.*, vol. 65, no. 15, pp. 3922–3933, Aug. 2017.

[20] R. Rajashekar and L. Hanzo, "Hybrid beamforming in mm-Wave MIMO systems having a finite input alphabet," *IEEE Trans. Commun.*, vol. 64, no. 8, pp. 3337–3349, Aug. 2016.

[21] X. Gao, L. Dai, C. Yuen, and Z. Wang, "Turbo-like beamforming based on tabu search algorithm for millimeter-wave massive MIMO systems," *IEEE Trans. Veh. Technol.*, vol. 65, no. 7, pp. 5731–5737, Jul. 2016.

[22] C.-B. Chae, D. Mazzarese, T. Inoue, and R. Heath, "Coordinated beamforming for the multiuser MIMO broadcast channel with limited feedforward," *IEEE Trans. Signal Process.*, vol. 56, no. 12, pp. 6044–6056, Dec. 2008.

[23] J. Choi, "Beam selection in mm-Wave multiuser MIMO systems using compressive sensing," *IEEE Trans. Commun.*, vol. 63, no. 8, pp. 2936–2947, Aug. 2015.

[24] W. Rhee, W. Yu, and J. M. Cioffi, "The optimality of beamforming in uplink multiuser wireless systems," *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 86–96, Jan. 2004.

[25] G. Lee, Y. Sung, and J. Seo, "Randomly-directional beamforming in millimeter-wave multiuser MISO downlink," *IEEE Trans. Wireless Commun.*, vol. 15, no. 2, pp. 1086–1100, Feb. 2016.

[26] X. Zhu, Z. Wang, L. Dai, and Q. Wang, "Adaptive hybrid precoding for multiuser massive MIMO." *IEEE Commun. Lett.*, vol. 20, no. 4, pp. 776–779, Apr. 2016.

[27] A. Alkhateeb, G. Leus, and R. W. Heath, "Limited feedback hybrid precoding for multi-user millimeter wave systems," *IEEE Trans. Wireless Commun.*, vol. 14, no. 11, pp. 6481–6494, Nov. 2015.

[28] J. Choi, "Analog beamforming for low-complexity multiuser detection in mm-Wave systems," *IEEE Trans. Veh. Technol.*, vol. 65, no. 8, pp. 6747–6752, Aug. 2016.

[29] J. Li, L. Xiao, X. Xu, and S. Zhou, "Robust and low complexity hybrid beamforming for uplink multiuser mmwave MIMO systems," *IEEE Commun. Lett.*, vol. 20, no. 6, pp. 1140–1143, Jun. 2016.

[30] A. Alkhateeb, O. El Ayach, G. Leus, and R. W. Heath, Jr., "Channel estimation and hybrid precoding for millimeter wave cellular systems," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 831–846, Oct. 2014.

[31] Z. Gao, C. Hu, L. Dai, and Z. Wang, "Channel estimation for millimeter-wave massive MIMO with hybrid precoding over frequency-selective fading channels," *IEEE Commun. Lett.*, vol. 20, no. 6, pp. 1259–1262, Jun. 2016.

[32] Z. Wang, M. Li, X. Tian, and Q. Liu, "Iterative hybrid precoder and combiner design for mmwave multiuser MIMO systems," *IEEE Commun. Lett.*, vol. 21, no. 7, pp. 1581–1584, Jul. 2017.

[33] A. Goldsmith, *Wireless Communications.* Cambridge, U.K.: Cambridge Univ. Press, 2005.

P. Raviteja received the B.Tech degree in electronics and communication from JNTU university, Hyderabad, India in 2012, and the M.E. degree in telecommunications from Indian Institute of Science, Bangalore, India in 2014. He is currently working toward the Ph.D. degree with the Department of Electrical and Computer Systems Engineering, Monash University, Clayton, VIC, Australia. From 2014 to 2015, he worked at Qualcomm India Private Limited, Bangalore, India on WLAN systems design. His current research interests include analog and hybrid beamforming design for millimeter wave communication systems. He is a recipient of the Prof. S. V. C. Aiya Medal from the Indian Institute of Science, 2014.

Yi Hong (S'00–M'05–SM'10) received the Ph.D. degree in electrical engineering and telecommunications from the University of New South Wales (UNSW), Sydney, NSW, Australia. She is currently a Senior Lecturer with the Department of Electrical and Computer Systems Engineering, Monash University, Clayton, VIC, Australia. She received the NICTA-ACoRN Earlier Career Researcher Award at the Australian Communication Theory Workshop, Adelaide, SA, Australia, 2007. She was an Associate Editor for the IEEE WIRELESS COMMUNICATION LETTERS and *Transactions on Emerging Telecommunications Technologies*. She was the General Co-Chair for the IEEE INFORMATION THEORY WORKSHOP 2014, Hobart, TAS, Australia; the Technical Program Committee Chair of *Australian Communications Theory Workshop* 2011, Melbourne, VIC, Australia; and the Publicity Chair at the IEEE INFORMATION THEORY WORKSHOP 2009, Sicily, Italy. She was a Technical Program Committee member for many IEEE leading conferences. Her research interests include communication theory, coding and information theory with applications to telecommunication engineering.

Emanuele Viterbo (M'95–SM'04–F'11) received the Ph.D. degree in electrical engineering from the Politecnico di Torino, Torino, Italy, in 1995. He is currently a Professor with the Department of Electrical, Computer and Systems Engineering and an Associate Dean in graduate research with Monash University, Clayton, VIC, Australia. From 1990 to 1992, he was with the European Patent Office, The Hague, The Netherlands, as a Patent Examiner in the field of dynamic recording and error-control coding. Between 1995 and 1997, he held a Postdoctoral position in the Dipartimento di Elettronica of the Politecnico di Torino. In 1997–1998, he was a Postdoctoral Research Fellow with the Information Sciences Research Center, AT&T Research, Florham Park, NJ, USA. From 1998 to 2005, he worked as an Assistant Professor and then an Associate Professor with the Dipartimento di Elettronica, Politecnico di Torino. From 2006 to 2009, he worked with the Department of Electronics, Informatics and Systems, University of Calabria, Rende, Italy, as a Full Professor. He is currently an ISI Highly Cited Researcher, since 2009. His main research interests include lattice codes for the Gaussian and fading channels, algebraic coding theory, algebraic space-time coding, digital terrestrial television broadcasting, digital magnetic recording, and irregular sampling. He is currently an Associate Editor for the IEEE TRANSACTIONS ON INFORMATION THEORY, *European Transactions on Telecommunications,* and *Journal of Communications and Networks*, and Guest Editor for the IEEE JOURNAL OF SELECTED TOPICS IN SIGNAL PROCESSING: Special Issue Managing Complexity in Multiuser MIMO Systems. He was awarded a NATO Advanced Fellowship in 1997 from the Italian National Research Council.