Rotated Trellis Coded Lattices

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1 Introduction

Lattice sphere packings have been studied by mathematicians since the beginning of this century [1]. A lattice is a discrete group of the real $N$-dimensional space $\mathbb{R}^N$. A finite subset of a lattice, usually called a lattice constellation, is used as a linear modulation and can be viewed as a generalized Quadrature Amplitude Modulation. The densest integer lattices are known in dimensions up to 32 [1]. These lattices are matched to additive white Gaussian noise channels and exhibit a gain of 4 to 6dB in signal-to-noise ratio. This gain, due to the lattice density, disappears on Rayleigh fading channels where the performance is essentially related to the lattice diversity [2]. A block code can fill up to $\rho$ erasures if $\rho + 1 \leq d_{H,\text{min}}$, where $d_{H,\text{min}}$ is the minimum Hamming distance. In an similar manner, a lattice has an $L$-diversity order if the minimum Hamming distance between the lattice points is equal to $L$. For such a lattice constellation the bit error rate curves decrease linearly on a logarithmic plot, since $BER = K/SNR^L$. This diversity is very much increased by rotating the original lattice. The multidimensional rotation spreads the same information over many axes which creates the so called modulation diversity since all axes cannot be faded at the same time. Many techniques have been found recently [3] to compute multidimensional rotations with high diversity. In this paper we combine rotations with trellis coded modulation (TCM) to create a rotated trellis coded modulation (RTCM) matched to Rayleigh fading channels.

2 RTCM Encoding and Decoding

Two methods are available to map $m$ information bits into one $N$-dimensional point of an integer lattice constellation. The lattice encoding is done directly by multiplying $N$ integers with the lattice generator matrix or by partitioning a bidimensional QAM constellation and combining it with a binary encoder. To build the RTCM encoder, we start by partitioning a lattice into sublattices (cosets), we associate an encoder (a binary block encoder + a bidimensional QAM) to each coset and we add a binary convolutional code that selects the coset to be transmitted. Uncoded information bits select the point inside the coset. This encoding scheme is analog to that of Multidimensional TCMs [5]. Finally, the encoded point is rotated before being transmitted over a fading channel.
<table>
<thead>
<tr>
<th>Partition</th>
<th>Index</th>
<th>rate</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z^4/2D_4$</td>
<td>32</td>
<td>4/5</td>
<td>512</td>
</tr>
<tr>
<td>$D_4/2D_4$</td>
<td>16</td>
<td>3/4</td>
<td>512</td>
</tr>
<tr>
<td>$Z^8/E_8$</td>
<td>16</td>
<td>3/4</td>
<td>$2^{17}$</td>
</tr>
<tr>
<td>$E_8/RE_8$</td>
<td>16</td>
<td>3/4</td>
<td>$2^{17}$</td>
</tr>
<tr>
<td>$Z^{16}/H_{16}$</td>
<td>32</td>
<td>4/5</td>
<td>$2^{33}$</td>
</tr>
</tbody>
</table>

The above Table shows five possible RTCMs. The first column gives the quotient group as $\Lambda_2/\Lambda_1$ where $\Lambda_1$ is a sublattice of $\Lambda_2$. The number of cosets of $\Lambda_1$ is indicated in the second column. The last columns give the convolutional encoder rate and the constellation size for a spectral efficiency of 2 bits per dimension.

For example, the 8-dimensional RTCM $Z^8/E_8$ exhibits a gain of 7dB at $10^{-3}$ when compared to the same TCM without rotation. This RTCM has the same performance as a 16-dimensional rotation over the Rayleigh channel. The RTCM decoding combines a sphere-decoding algorithm [4] with a Viterbi algorithm. The sphere-decoder chooses the nearest lattice point and the Viterbi algorithm uses the squared distance between the received point and the decoded one to find the optimal trellis survivor.

3 Conclusions

For high spectral efficiency ($\geq 4$ bits per symbol), a powerful trellis coded modulation designed for the additive white Gaussian noise channel has a poor performance over the Rayleigh fading channel. This performance is largely improved by a multidimensional rotation that creates a high diversity order which is essential when transmitting over fading channels.

References


