Ray-Tracing Simulation of Cross-Road Scenarios Based on a Stochastic Model for Vehicular Traffic

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Abstract—Vehicle-to-infrastructure (V2I) and vehicle-to-vehicle (V2V) communications find extensive applications, particularly for congestion avoidance and road safety. However, development of such communication systems require accurate modeling of the wireless channel. This paper presents a methodology to simulate cross-road environments involving vehicular traffic. We propose a novel modeling approach for vehicular traffic based on a non-stationary one-dimensional Poisson arrival process which is represented by a $M/M/\infty$ queueing model. The joint ray-tracing of the stationary scatterers such as surrounding buildings along with stochastic modeling of mobile scatters such as vehicles reduces simulation time significantly, and yields relevant statistics of channel parameters with a minor compromise in accuracy.

Index terms— Channel modeling, ray-tracing, Doppler, vehicle-to-infrastructure (V2I).

I. INTRODUCTION

The advent of intelligent transportation systems has led to the emergence of vehicle-to-infrastructure (V2I) and vehicle-to-vehicle (V2V) communications [1]–[3] which is being extensively researched in the recent past. However, there are several challenges involved in the development of V2I and V2V communication systems primarily due to the dynamically varying channel statistics [4] encountered in application scenarios. Also, accounting for the non-stationarity of such channels is an important consideration in order to ensure error-free analysis of V2I and V2V communication systems. Therefore, accurate knowledge of channel statistics and the development of approaches to model realistic channels are the key to design such systems.

Channel models for V2I and V2V communication systems are largely classified as measurement based models [5], geometry based stochastic models [6], ray-tracing (deterministic) models [7], and a combination of measurement and ray-tracing models (quasi-deterministic models) [8], [9]. In measurement based channel models, channel parameters are modeled via probability density functions (PDF) derived from curve fitting of measurement data. Geometry based stochastic models assume a known location of scatterers and obtain channel parameters based on the geometry of the model. Both, measurement and geometry based channel models, account for the trade-off between accuracy and complexity of modeling V2I and V2V channels. However, certain applications such as deployment of infrastructure for V2I communications require accurate knowledge of site specific channel characteristics. In these scenarios, ray-tracing models which employ geometric optics and geometrical theory of diffraction based on the exact geometry of the propagation environment can be employed to generate channel features. In fact, ray-tracing models are popularly used to run accurate site-specific simulations, albeit at the cost of high computational complexity. The number of computations in ray-tracing simulations is exorbitant for V2I and V2V channels. This is due to the complexity involved in performing extensive ray-tracing simulations for all possible locations of vehicles present in the environment. In order to alleviate such prohibitive computational complexity, a combination of simplified ray tracing and stochastic based modeling (quasi-deterministic) has been previously explored [8], [9]. In this approach, dominant multi-path components such as line of sight (LOS) as well as those due to buildings are generated using ray-tracing simulations while multi-path components due to moving objects and relatively smaller objects such as sign boards are modeled using stochastic models without taking into consideration the non-stationarity of V2I and V2V channels.

Vehicular channels are known to exhibit gradual appearance and disappearance (birth and death) of multi-path components due to continuous motion of vehicles [10]. Moreover, non-uniform vehicular traffic introduces variations in velocity of vehicles which further alter the birth and death rate of multi-path components [11], [12]. A geometry based stochastic modeling approach which captures aforementioned characteristics of V2I and V2V channels is developed in [11] but it only accounts for changes in velocity of the mobile user (MU) under constant acceleration. The model does not provide insights in scenarios where location and velocity of MU can assume arbitrary values [12]. Moreover, existing literature lacks channel models which combine deterministic and stochastic aspects of modeling vehicular channels under varying vehicular velocities. In this paper, we propose a novel quasi-deterministic channel model to simulate V2I channels, specifically characterizing cross-roads. In the proposed modeling approach, the birth-death events of multi-path components in V2I channels are modeled using an $M/M/\infty$ queue [10], whereas LOS as well as multi-path components due to surrounding buildings are generated using ray-optics. Moreover, in order to characterize non-uniform traffic near cross-roads, we also introduce variable birth and death rate for the generation of multi-path components due to vehicles.

The rest of the paper is organized as follows: Section II introduces the system model for the proposed quasi-
The proposed stochastic approach for modeling multi-path components due to non-uniform vehicular traffic is elaborated in Section III. Section IV presents numerical simulations. Finally, Section V concludes the paper.

II. QUASI-DETERMINISTIC APPROACH FOR MODELING V2I CHANNELS

We consider a cross-road scenario as depicted in Fig. 1 to simulate the V2I channel. For simplicity, we assume a 2D propagation scenario in order to introduce the proposed quasi-deterministic channel model. It might be noted that the proposed approach can easily be extended to model more realistic propagation environments by modifying the ray-tracing part of the model. In this model, we assume that a MU represented by a red rectangle is moving with a time-variant velocity \( \vec{v}(t) \) in an arbitrary direction, and is served by one of the base stations (BSs) indicated by a red and yellow discs as depicted in Fig. 1. The BS and MU are equipped with omni-directional antennas. We assume that other moving vehicles also concurrently present in the environment. The multi-path channel between the BS and MU is assumed to be influenced by the stationary scatterers (for e.g., buildings) as well as moving vehicles.

![Fig. 1. A typical scenario for V2I communications.](image)

The proposed quasi-deterministic channel model is characterized by the time-varying impulse response given in (1). The parameters, \( L_D(t) \) and \( L_R(t) \), in (1) represent the instantaneous number of multi-path components received at time \( t \) due to buildings and vehicles, respectively. \( \alpha_{D,l} \) and \( \alpha_{R,l} \) are the complex-valued gain of \( l \)-th multi-path components. The instantaneous velocity of the \( l \)-th scatterer (associated to a vehicle) relative to MU is denoted by \( v_{R,l}(t) \). The variables, \( \varphi_{D,l}(t) = \int_0^t k \pi(t') \cos(\phi_{D,l}(t')) dt' \) and \( \varphi_{R,l}(t) = \int_0^t k v_{R,l}(t') \cos(\phi_{R,l}(t')) dt' \) are the time-variant phases caused by Doppler frequency changes due to buildings and vehicles, respectively. \( k \) denotes wave number, and \( \phi_{D,l}(t) \) and \( \phi_{R,l}(t) \) are angle of arrival for multi-path components.

\[
h(t) = h_D(t) + h_R(t) = \sum_{l=1}^{L_D(t)} \alpha_{D,l}(t) \exp(j \varphi_{D,l}(t)) \delta(t - \tau_{D,l}) + \sum_{l=1}^{L_R(t)} \alpha_{R,l}(t) \exp(j \varphi_{R,l}(t)) \delta(t - \tau_{R,l}),
\]  

(1)

Temporal fluctuations of propagation channels manifest in terms of the appearance of multi-path components and their disappearance after a finite duration of time. One such scenario is illustrated in Fig. 2. As vehicles continue to move in their respective directions, the multi-path profile of the channel begins to change. For example, the MU initially receives a multi-path component due to a vehicle moving in close proximity as shown in Fig 2(a). After a certain amount of time elapses, as shown in Fig 2(b), the MU begins to receive an additional multi-path component which is coming from a different direction. The number of multi-path components received with as a function of time for this scenario is shown in Fig 2(c). This characteristics of such time varying channels can generally be modeled using \( M/M/\infty \) queuing models [10], [13]. Following this approach, the appearance of multi-path components is modeled using a Poisson arrival process with rate \( \lambda \), and their disappearance by another Poisson process with an average life time, \( 1/\mu \). During the appearance of a new multi-path component, the corresponding delay, directions, and path gain are synthesized from predefined PDFs. Addition-

Fig. 2. Birth and death process: (a) snapshot of the environment at time \( t_1 \), (b) snapshot of the environment at time \( t_2 \), (c) total number of multi-path components due to buildings and vehicles.
ally, the spatio-temporal parameters of generated multi-path components such as delay, angular profile, and path gain may be altered before their disappearance. These characteristics are modeled via the technique called inheritance [13]. The updated values of delay and directions for a specified multi-path component can be inherited based on the estimated location of scatterers [13].

In the proposed modeling approach, we generate the parameter, $L_R(t)$ in terms of the queue length of $M/M/\infty$ at time instant $t$. $M/M/\infty$ is characterized by the time-varying arrival rate, $\lambda(t)$, and service rate, $\mu(t)$. We choose suitable functions for $\lambda(t)$ and $\mu(t)$ based on location dependent vehicular traffic in the environment. It should be noted that vehicular traffic would be high in close proximity to the cross-road as compared to other parts of the road. High vehicular traffic at cross-roads may cause a reduction of $\lambda(t)$ and $\mu(t)$ owing to the fact that vehicles near to MU are slowly moving in its vicinity. On the other hand, if the MU is at a farther distance from the cross-road, the arrival rate, $\lambda(t)$, and service rate, $\mu(t)$ are expected to be high due to frequent passage of vehicles. The instantaneous values $\lambda(t)$ and $\mu(t)$ are evaluated based on the location of MU at time instant $t$. Once the number of multi-path components $L_R(t)$ is determined based on the proposed $M/M/\infty$ queuing model, we find the parameters $\alpha_{R,l}(t)$, $\tau_{R,l}(t)$, and $\phi_{R,l}(t)$ based on the procedure described in the following section.

### III. Modeling Parameters $\alpha_{R,l}(t)$, $\tau_{R,l}(t)$, and $\phi_{R,l}(t)$

The parameters $\tau_{R,l}(t)$, $\alpha_{R,l}(t)$, and $\phi_{R,l}(t)$ can be found using PDFs. The delay parameter, $\tau_{R,l}$, is generated following the approach outlined in [13]. Let $d(t)$ denote the Euclidean distance between BS and the MU when a new multi-path component is received at the MU. The propagation delay of this multi-path component is given by,

$$\tau_{R,l}(t) = \tau_{R,l}(0) + c^{-1}d(t), \quad (2)$$

where $c$ denotes the velocity of light and $\tau_{R,l}'$ represents excess delay which is uniformly distributed in the interval $[0, \tau_m]$. The inherited time delay for $l$-th multi-path component after a duration $\Delta t$ is given by [13], $\tau_{R,l}(t+\Delta t) = \tau_{R,l}(t) + \Delta \tau_{R,l}(t)$, where $\Delta \tau_{R,l}(t) = \pm \frac{1}{2} \Delta d(t)$ with $\Delta d(t)$ denotes the incremental change in position of MU during the time interval $\Delta t$. $\Delta \tau_{R,l}$ is positive if the user moves towards the BS, and negative, otherwise. Angle-of-arrival, $\phi_{R,l}$ of the newly generated multi-path component, represented by the index, $l$, is synthesized using Von Mises PDF, i.e., $\phi_{R,l}(t) \sim \exp(k \cos(\phi_{R,l}(t) - \nu))/2mI_0(k_0)$, where $\phi_{R,l} \in [-\pi, \pi]$, $I_0(.)$ is the zeroth-order modified Bessel Function of the first kind, $\nu$ denotes mean value of $\phi_{R,l}$. Angle-of-arrival of the $l$-th multi-path component after the duration $\Delta t$ is inherited from $\phi_{R,l}(t)$ using the relationship,

$$\phi_{R,l}(t+\Delta t) = \phi_{R,l}(t) + \Delta \phi_{R,l}(t), \quad (3)$$

where $\Delta \phi_{R,l}(t)$ is assumed to follow a normal distribution [13], i.e., $\Delta \phi_{R,l}(t) \sim \mathcal{N}(0, \sigma^2)$. The path gain is given by $\alpha_{R,l} = \sqrt{C_{R,l}} \Gamma_r(c \tau_{R,l})^{-\gamma}$, where $C_{R,l}$ is the path loss intercept point, $\Gamma_r$ is the average reflection loss and the parameter $\gamma$ denotes the path loss exponent. Consistent with existing channel models [6], [11], the initial phase angle, $\psi_{R,l}$ is assumed to be uniformly distributed over the interval $[0, 2\pi]$.

The velocity of the $l$-th multi-path component is generally correlated with the duration over which the multi-path is present in the received signal (life time of the multi-path component). This implies that a given multi-path component remains present in received signal for a longer duration if the relative velocity between the vehicle and the MU is less, and vice-versa. Therefore, we map the average life time of a multi-path component, $1/\mu(t)$ to $v_{R,l}(t)$ as $v_{R,l}(t) = K \mu(t)$, where $K$ is a proportionality constant which depends on the electrical length of the scatterer (i.e., the vehicle causing the multi-path component). We also assume that the relative changes in velocity of a vehicle are negligible as compared to the life time of the corresponding multi-path component, i.e., $v_{R,l}(t+\Delta t) \approx v_{R,l}(t)$. Based on the mathematical framework developed in this section, we propose the quasi-deterministic channel model for simulating V2I channels described in Algorithm 1.

#### Algorithm 1 Quasi-deterministic channel model

1. Initialize: Generate time samples, $t_i$ from $[0, T_{max}]$, initial position of MU $[x(t_0)]$
2. for each $i \in \{1, 2, \ldots , N\}$ do
3. Determine the current position of MU, $x(t_i)$ based on $x(t_{i-1}), \tau(t_i)$, and its direction of motion.
4. Determine $\lambda(t_i)$ and $\mu(t_i)$ based on the current position of MU, $x(t_i)$.
5. Determine $h_D(t_i)$ using a ray-tracing simulation.
6. Determine $h_R(t_i)$ based on the stochastic model described in Section III.
7. Combine $h_D(t_i)$ and $h_R(t_i)$ as given in (1) and obtain $h(t_i)$.

![Fig. 3. An example of modeling time varying quantities $\tau(t)$, $\lambda(t)$, and $\mu(t)$.](image)

### IV. Numerical Results

We carry out numerical simulations over the deployment area illustrated in Fig. 1 in order to determine the time varying impulse response of the V2I channel. The origin, $(0,0)$ coincides with the center point of the cross road; each roads is assumed to have a width of 6 m. The location of the BS 1 and BS 2 are arbitrarily chosen at $(2.8, 4)$ and $(5.2, 8)$, respectively. We assume that the MU starts moving from left end of the road and continues along the same road towards its right end. To demonstrate the proposed channel model, we first characterize the location dependent velocity of MU, multi-path arrival rate, and service rate using a simple approach. By
mapping between time and spatial coordinates, $\tau(t)$, $\lambda(t)$, and $\mu(t)$ of multi-path components are modeled using a piece-wise function as depicted in Fig 3. Here, $t_2$ and $t_3$ can be considered as the time instants at which the MU reaches at the intersection of edges of the two crossing roads. Moreover, we assume that $f(t)$ starts to decrease after the time instant $t_1$, since the vehicle is approaching the cross road, whereas $f(t)$ begins to increase after time instant $t_2$ since the vehicle is receding from the cross road. This simple but realistic model ensures that $v(t_0)$, $\lambda(t)$, and $\mu(t)$ are low at the cross road as compared to other regions of the road owing to the high vehicular traffic near the cross road. Notably, other suitable models for $v(t_0)$, $\lambda(t)$, and $\mu(t)$ can be adopted to mimic more realistic vehicular traffic patterns. In order to ensure spatial consistency of $\tau(t)$, $\lambda(t)$, and $\mu(t)$, the spatial location of the MU must be mapped properly with time. We consider that the time scale is sampled within a sufficiently small interval so that the variation of velocity over the interval is negligible. Hence, the modified location of the user node at sample instance $t_i$ is given by, $x(t_i) = x(t_{i-1}) - \Delta t \tau(t_{i-1})$, where $\Delta t$ is the sampling interval. We choose maximum and minimum values of $\tau(t)$, $\lambda(t)$, and $\mu(t)$ in accordance with $f(t)$ as: $\tau_m = 30$ km/hr, $\tau_0 = 15$ km/hr, $\lambda_m = 2\tau_m/K$, $\lambda_0 = 2\tau_0/K$, $\mu_m = \lambda_m$, and $\mu_0 = \lambda_0$. Other parameters take values, $\Gamma_r = -3$ dB, $\gamma = 2$, $T_{\text{max}} = 5$ s, $k_1 = 2$, $\Delta = 0.125$ m, $\sigma_0 = 5^0$, and $\tau_m = 1.84 \times 10^{-7}$ s [13]. In this paper, we conduct ray tracing of the V2I channel based on the framework developed in [14]. Direction, path gain, and propagation delay of reflection components from building walls are found using ray optics at each position of MU, $x(t_i)$.

The distribution of multi-path components in the Doppler-delay domain for different time instants is shown in Fig. 4. Pictures depicting location of the MU corresponding to the snapshots of channel is also included in Fig. 4. As the MU continues to move, multi-path components appear and disappear after being present in the received signal for a brief duration of time depending on the velocity of vehicles including the MU. We then plot the signal power received from BS 1 and BS 2, calculated based on the proposed channel model (with unit transmit power) simulated for a duration of 5 s in Fig. 5(a). For BS 1, as the MU starts traveling towards cross-road, it experiences a non-LOS condition since the LOS component is blocked by an adjacent building in this case, and consequently the received signal incurs a severe path loss. As the MU approaches closer to the cross road, it starts receiving the LOS component from BS 1 and path loss is considerably reduced. It can be seen that at around the time instant, 1.92 s, the MU leaves the cross road and the communication channel between the MS and the BS once again becomes non-LOS due to the of blockage of the LOS component leading to a rapid fall in the received signal power. For the signal from BS 2, the MS always experiences LOS condition with maximum power is received when it is close to BS 2. Temporal autocorrelation function corresponding to the channel gain between BS 2 and MU, sampled at two time instances is plotted in Fig. 5(b) (corresponding locations of the MS are also shown). At time instance $t = 1$ s, the MS is located inside the cross-road and experiences slow changes in channel gain due to low velocity of vehicles. However, at time instance $t = 4$ s, the MS leaves the cross road. High velocity of vehicles introduces fast variations in channel gain as indicated by the rapidly reducing autocorrelation function plotted in Fig. 5(b). Corresponding time-variant power spectral density (PSD) of channel is shown in Fig. 5(c). The Doppler PSD evaluated at $t = 1$ is appeared to be more specular than
the spectrum at $t = 4$ s due to the presence of dominant LOS component. Moreover, the peak of the Doppler PSDs move to higher values when MS leaves cross-road, owing to the fact that the velocity of vehicles begins to increase.

V. CONCLUSION

In this paper, we develop a novel computationally efficient channel model to conduct site-specific simulation of V2I communication channels. Joint ray-tracing of multi-path components originating from surrounding buildings and stochastic modeling of multi-path components generated due to moving vehicles enables site-specific simulation of V2I channels with significantly reduced computational complexity as compared to conventional ray-tracing tools. We demonstrate the proof of concept through extensive simulations. As a case study, environment dependent received signal power and time varying Doppler power spectral density of V2I channels in a typical cross-road environment are analyzed. The results indicate that the location dependent vehicular traffic can cause major changes on the statistics of channel. Thus, the proposed channel model can be applied to investigate the performance of orthogonal frequency division multiplexing and orthogonal time frequency modulations based communication systems.

As future scope, the proposed channel model can be extended to model scenarios where moving vehicles influence the properties of multi-path components generated by buildings. Moreover, on-site channel measurements similar to [15] can be conducted in order to validate the proposed channel modeling approach.

ACKNOWLEDGEMENT

This work was supported by the Australian Research Council through the Discovery Project under Grant DP160100528.

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